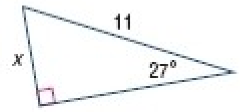
Study Guide and Review - Chapter 4

Find the value of *x*. Round to the nearest tenth, if necessary.

13. 

SOLUTION:

An acute angle measure and the length of the hypotenuse are given, so the sine function can be used to find the length of the side opposite θ.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$
$$\sin 27° = \frac{x}{11}$$
$$11\sin 27° = x$$
$$5.0 \approx x$$

Find the measure of angle *θ*. Round to the nearest degree, if necessary.

15. 

SOLUTION:

Because the length of the side opposite θ and the hypotenuse are given, use the sine function.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$
$$\sin\theta = \frac{6}{18}$$
$$\theta = \sin^{-1}\frac{6}{18} \text{ or about } 19°$$

Write each degree measure in radians as a multiple of π and each radian measure in degrees.

17. 135°

SOLUTION:

To convert a degree measure to radians, multiply by $\frac{\pi \text{ radians}}{180°}$.

$$135° = 135°\left(\frac{\pi \text{ radians}}{180°}\right)$$
$$= \frac{3\pi}{4} \text{ radians or } \frac{3\pi}{4}$$

19. $\frac{7ð}{4}$

SOLUTION:

To convert a radian measure to degrees, multiply by $\frac{180°}{\pi \text{ radians}}$.

$$\begin{aligned}\frac{7\pi}{4} &= \frac{7\pi}{4} \text{ radians} \\ &= \frac{7\pi}{4} \text{ radians}\left(\frac{180°}{\pi \text{ radians}}\right) \\ &= \frac{7(180°)}{4} \\ &= 315°\end{aligned}$$

Identify all angles coterminal with the given angle. Then find and draw one positive and one negative angle coterminal with the given angle.

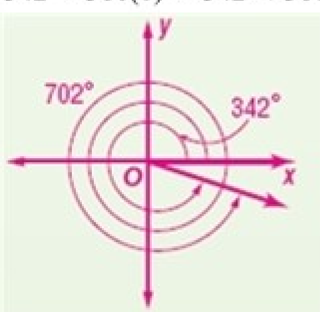
21. 342°

SOLUTION:

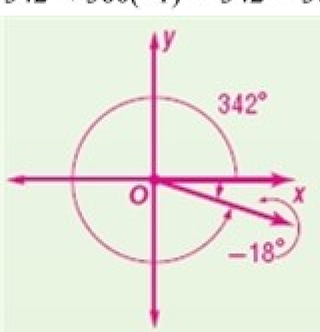
All angles measuring $342° + 360n°$ are coterminal with a 342° angle.

Sample answer: Let n = 1 and −1.

$342° + 360(1)° = 342° + 360° \text{ or } 702°$



$342° + 360(-1)° = 342° - 360° \text{ or } -18°$



Find the area of each sector.

23. 

SOLUTION:

The measure of the sector's central angle θ is 31° and the radius is 14 inches. Convert the central angle measure to radians.

$$31° = 31°\left(\frac{\pi \text{ radians}}{180°}\right) = \frac{31\pi}{180}$$

Use the central angle and the radius to find the area of the sector.

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}(14)^2\left(\frac{31\pi}{180}\right) \approx 53.0$$

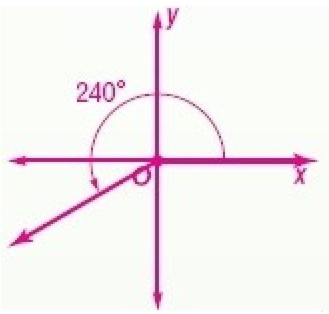
Therefore, the area of the sector is about 53.0 square inches.

Sketch each angle. Then find its reference angle.

25. $240°$

SOLUTION:

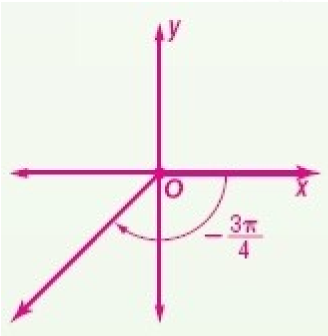
The terminal side of 240º lies in Quadrant III. Therefore, its reference angle is $\theta' = 240° - 180°$ or 60º.



27. $-\frac{3\pi}{4}$

SOLUTION:

The terminal side of $-\frac{3\pi}{4}$ lies in Quadrant III. Therefore, its reference angle is $\theta' = \pi - \frac{3\pi}{4}$ or $\frac{\pi}{4}$.



Find the exact values of the five remaining trigonometric functions of θ.

29. $\cos\theta = \frac{2}{5}$, where $\sin\theta > 0$ and $\tan\theta > 0$

SOLUTION:

To find the other function values, find the coordinates of a point on the terminal side of θ. Because $\sin\theta$ and $\tan\theta$ are positive, θ must lie in Quadrant I. This means that both x and y are positive.

Because $\cos\theta = \frac{x}{r}$ or $\frac{2}{5}$, $x = 2$ and $r = 5$. Find y.

$$\begin{aligned} y &= \sqrt{r^2 - x^2} \\ &= \sqrt{5^2 - 2^2} \\ &= \sqrt{21} \end{aligned}$$

Use $x = 2$, $y = \sqrt{21}$, and $r = 5$ to write the five remaining trigonometric ratios.

$$\sin\theta = \frac{y}{r} \text{ or } \frac{\sqrt{21}}{5} \qquad \csc\theta = \frac{r}{y} = \frac{5}{\sqrt{21}} \text{ or } \frac{5\sqrt{21}}{21}$$

$$\tan\theta = \frac{y}{x} \text{ or } \frac{\sqrt{21}}{2} \qquad \cot\theta = \frac{x}{y} = \frac{2}{\sqrt{21}} \text{ or } \frac{2\sqrt{21}}{21}$$

$$\sec\theta = \frac{r}{x} \text{ or } \frac{5}{2}$$

31. $\sin\theta = -\frac{5}{13}$, where cos θ > 0 and cot θ < 0

SOLUTION:

$\sin\theta = -\frac{5}{13}$, where cos θ > 0 and cot θ < 0

To find the other function values, find the coordinates of a point on the terminal side of θ. Because cos θ is positive and cot θ is negative, θ must lie in Quadrant IV. This means that x is positive and y is negative.

Because sin θ = $\frac{y}{r}$ or $-\frac{5}{13}$, y = –5 and r = 13. Find x.

$$\begin{aligned} x &= \sqrt{r^2 - y^2} \\ &= \sqrt{13^2 - (-5)^2} \\ &= \sqrt{144} \text{ or } 12 \end{aligned}$$

Use x = 12, y = –5, and r = 13 to write the five remaining trigonometric ratios.

$$\cos\theta = \frac{x}{r} \text{ or } \frac{12}{13} \qquad \sec\theta = \frac{x}{r} \text{ or } \frac{13}{12}$$

$$\tan\theta = \frac{y}{x} \text{ or } -\frac{5}{12} \qquad \cot\theta = \frac{x}{y} \text{ or } -\frac{12}{5}$$

$$\csc\theta = \frac{r}{y} \text{ or } -\frac{13}{5}$$

Find the exact value of each expression. If undefined, write *undefined*.

33. sin 180°

SOLUTION:

Because the terminal side of θ lies on the negative x-axis, the reference angle θ ' is π.

$$\sin\frac{0}{1} = 0$$

35. sec 450°

SOLUTION:

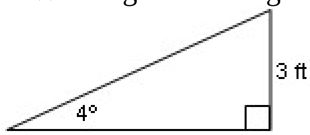
Because the terminal side of θ lies on the positive y-axis, the reference angle θ ' is $90°$.

$$\begin{aligned} \sec 450° &= \sec 90° \\ &= \frac{1}{0} \text{ or undefined} \end{aligned}$$

67. **CONSTRUCTION** A construction company is installing a three-foot-high wheelchair ramp onto a landing outside of an office. The angle of the ramp must be 4°.
a. What is the length of the ramp?
b. What is the slope of the ramp?

SOLUTION:

a. We are given the angle and the height opposite the angle. Draw a diagram.



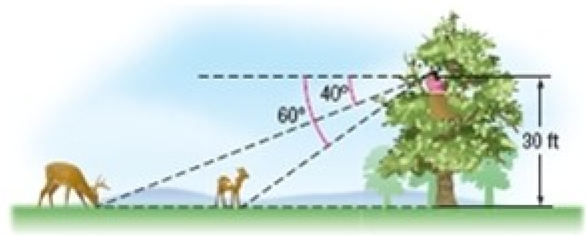
We need to find the length of the ramp, so we can use the tangent function.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 4 = \frac{3}{x}$$
$$x\tan 4 = 3$$
$$x = \frac{3}{\tan 4}$$
$$x \approx 42.9$$

b. The slope is equal to the rise over the run. One coordinate is (0, 0) and another coordinate is (43, 3).

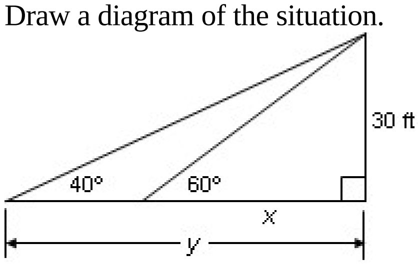
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{3-0}{43-0}$$
$$= \frac{3}{43}$$
$$\approx 0.07$$

68. **NATURE** For a photography project, Maria is photographing deer from a tree stand. From her sight 30 feet above the ground, she spots two deer in a straight line, as shown below. How much farther away is the second deer than the first?



SOLUTION:

Draw a diagram of the situation.



The 40° and 60° angles shown are equal to the angles in the original picture because they are alternate interior angles.

We can use trigonometry to find x and y and then find $y - x$, which is the distance between the deer.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 60 = \frac{30}{x}$$

$$x\tan 60 = 30$$

$$x = \frac{30}{\tan 60}$$

$$\tan 40 = \frac{30}{y}$$

$$y\tan 40 = 30$$

$$y = \frac{30}{\tan 40}$$

$$y - x = \frac{30}{\tan 40} - \frac{30}{\tan 60}$$

$$y - x \approx 18.4$$

69. **FIGURE SKATING** An Olympic ice skater performs a routine in which she jumps in the air for 2.4 seconds while spinning 3 full revolutions.

a. Find the angular speed of the figure skater.

b. Express the angular speed of the figure skater in degrees per minute.

SOLUTION:

a. The rate at which an object rotates about a fixed point is its angular speed. The angular speed is given by $\omega = \frac{\theta}{t}$.

Each rotation requires 2π radians, so θ = 6π.

$$\omega = \frac{\theta}{t} = \frac{6\pi}{2.4} = 2.5\pi$$

The angular speed is 2.5π radians per second.

b. Convert the seconds to minutes then convert the radians to degrees.

$$\frac{2.5\pi \text{ radians}}{\text{second}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} = \frac{150\pi \text{ radians}}{\text{minute}}$$

$$\frac{150\pi \text{ radians}}{\text{minute}} \cdot \frac{180 \text{ degrees}}{\text{radian}} = \frac{27,000 \text{ degrees}}{\text{minute}}$$

The angular speed is about 27,000 degrees per minute.

71. **WORLD'S FAIR** The first Ferris wheel had a diameter of 250 feet and took 10 minutes to complete one full revolution.
a. How many degrees would the Ferris wheel rotate in 100 seconds?
b. How far has a person traveled if she has been on the Ferris wheel for 7 minutes?
c. How long would it take for a person to travel 200 feet?

SOLUTION:

a. One complete revolution is done in 10 minutes, so divide 100 seconds by 10 minutes to determine the fraction of a revolution that has been completed in 100 seconds.

$$10 \text{ minutes} = 600 \text{ seconds}$$
$$\frac{100}{600} = \frac{1}{6}$$

One sixth of a revolution is equal to $\frac{360}{6}$ or 60 degrees.

b. If someone has been on it for 7 minutes, then they have made $\frac{7}{10}$ of a revolution. The wheel has a diameter of 250 feet, so the circumference is 250π feet. After 7 minutes, $\frac{7}{10}$ of the circumference has been traveled.

$$\frac{7}{10}(250\pi) = 175\pi$$
$$\approx 550$$

They have traveled about 550 feet.

c. It takes 10 minutes to travel the circumference, or 250π feet. Use a proportion to solve for *t*.

$$\frac{250\pi}{10} = \frac{200}{t}$$
$$2000 = 250\pi t$$
$$\frac{2000}{250\pi} = t$$
$$\frac{8}{\pi} = t$$
$$2.5 \approx t$$

It would take about 2.5 minutes to travel 200 feet.