Study Guide and Review - Chapter 3

Evaluate each expression.

21. $\text{Log}_2 32$

SOLUTION:

$$\begin{aligned}\log_2 32 &= \log_2 2^5 \\ &= 5\end{aligned}$$

ANSWER:

5

23. $\log_{25} 5$

SOLUTION:

$$\begin{aligned}\log_{25} 5 &= \log_{25} 25^{\frac{1}{2}} \\ &= \frac{1}{2}\end{aligned}$$

ANSWER:

$\frac{1}{2}$

25. $\ln e^{11}$

SOLUTION:

$$\begin{aligned}\ln e^{11} &= \log_e e^{11} \\ &= 11\end{aligned}$$

ANSWER:

11

27. $\log 80$

SOLUTION:

$\log 80 \approx 1.90$

ANSWER:

≈ 1.90

Expand each expression.

32. $\text{Log}_3\ 9x^3y^3z^6$

SOLUTION:

$$\begin{aligned}\log_3 9x^3y^3z^6 &= \log_3 9 + \log_3 x^3 + \log_3 y^3 + \log_3 z^6 \\ &= 2 + 3\log_3 x + 3\log_3 y + 6\log_3 z\end{aligned}$$

ANSWER:

$2 + 3 \log_3 x + 3 \log_3 y + 6 \log_3 z$

34. $\ln \dfrac{e}{x2y3z}$

SOLUTION:

$$\begin{aligned}\ln\frac{e}{x^2y^3z} &= \ln e - (\ln x^2 + \ln y^3 + \ln z) \\ &= 1 - (2\ln x + 3\ln y + \ln z) \\ &= 1 - 2\ln x - 3\ln y - \ln z\end{aligned}$$

ANSWER:

$1 - 2 \ln x - 3 \ln y - \ln z$

Condense each expression.

36. $3 \log_3 x - 2 \log_3 y$

SOLUTION:

$$\begin{aligned}3\log_3 x - 2\log_3 y &= \log_3 x^3 - \log_3 y^2 \\ &= \log_3 \frac{x^3}{y^2}\end{aligned}$$

ANSWER:

$\log_3 \dfrac{x3}{y2}$

38. $5 \ln (x + 3) + 3 \ln 2x - 4 \ln (x - 1)$

SOLUTION:

$$\begin{aligned} 5\ln(x+3)+3\ln 2x-4\ln(x-1) &= \ln(x+3)^5+\ln(2x)^3-\ln(x-1)^4 \\ &= \ln(x+3)^5+\ln 8x^3-\ln(x-1)^4 \\ &= \ln\frac{8x^3(x+3)^5}{(x-1)^4} \end{aligned}$$

ANSWER:

$$\ln \frac{8x3(x+3)5}{(x1)4}$$

Solve each equation.

39. $3^{x+3} = 27^{x-2}$

SOLUTION:

$$\begin{aligned} 3^{x+3} &= 27^{x-2} \\ 3^{x+3} &= (3^3)^{x-2} \\ 3^{x+3} &= 3^{3x-6} \\ x+3 &= 3x-6 \\ 9 &= 2x \\ \frac{9}{2} &= x \end{aligned}$$

ANSWER:

$$\frac{9}{2}$$

40. $25^{3x+2} = 125$

SOLUTION:

$$\begin{aligned} 3^{x+3} &= 27^{x-2} \\ 25^{3x+2} &= 125 \\ (5^2)^{3x+2} &= 5^3 \\ 5^{6x+4} &= 5^3 \\ 6x+4 &= 3 \\ 6x &= -1 \\ x &= -\frac{1}{6} \end{aligned}$$

ANSWER:

$$-\frac{1}{6}$$

41. $e^{2x} - 8e^{x} + 15 = 0$

SOLUTION:

$$\begin{aligned} e^{2x} - 8e^{x} + 15 &= 0 \\ (e^{x} - 5)(e^{x} - 3) &= 0 \\ e^{x} - 5 &= 0 \\ e^{x} &= 5 \\ x &= \ln 5 \\ e^{x} - 3 &= 0 \\ e^{x} &= 3 \\ x &= \ln 3 \end{aligned}$$

$x = \ln 3$ or $\ln 5$.

ANSWER:

$\ln 3; \ln 5$

42. $e^{x} - 4e^{-x} = 0$

SOLUTION:

$$\begin{aligned} e^{x} - 4e^{-x} &= 0 \\ e^{2x} - 4 &= 0 \\ e^{2x} &= 4 \\ \ln e^{2x} &= \ln 4 \\ 2x &= \ln 4 \\ x &= \frac{\ln 4}{2} \\ x &= \frac{\ln 2^{2}}{2} \\ x &= \frac{2 \ln 2}{2} \\ x &= \ln 2 \end{aligned}$$

ANSWER:

$\ln 2$

43. $\log_2 x + \log_2 3 = \log_2 18$

SOLUTION:

$$\begin{aligned}\log_2 x + \log_2 3 &= \log_2 18\\ \log_2 3x &= \log_2 18\\ 3x &= 18\\ x &= 6\end{aligned}$$

ANSWER:

6

44. $\log_6 x + \log_6 (x - 5) = 2$

SOLUTION:

$$\begin{aligned}\log_6 x + \log_6 (x-5) &= 2\\ \log_6 [x(x-5)] &= 2\\ x(x-5) &= 6^2\\ x^2 - 5x &= 36\\ x^2 - 5x - 36 &= 0\\ (x-9)(x+4) &= 0\\ x &= 9 \text{ or } 4\end{aligned}$$

$4 - 5 < 0$ and the logarithm of a negative number provides no real solution, so $x = 9$.

ANSWER:

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