

Chapter 5: Trigonometric Identities and Equations

Topics: Identify and use basic trigonometric identities: reciprocal, quotient, and Pythagorean, cofunction and odd-even identities. Simplify and rewrite trig expressions. Verify trig identities by combining fractions, multiplying, or factoring. Solve trig equations by isolating expressions, taking the square root of both sides, factoring, squaring both sides, and using trig identities to rewrite to then solve. Use sum and difference identities to evaluate trig expressions for angles not on the unit circle, rewrite an expression that has the form of a sum or difference identity as a single trig expression, evaluate different identities, and solve trig equations.

1. List the following identities:

- a. reciprocal (6) b. quotient (2) c. pythagorean (3)

2. Simplify:

- a. $\csc x \tan x$ to $\sec x$ b. $\cos^2 \theta \csc \theta \sec \theta$ to $\cot \theta$ c. $\frac{1 - \cos^2 x}{\tan x}$ to $\sin x \cos x$

3. Verify:

- a. $\sec x(\sec x - \cos x) = \tan^2 x$ b. $\csc^2 \theta - \cos^2 \theta \csc^2 \theta = 1$ c. $(\sec x + 1)(\sec x - 1) = \tan^2 x$
d. $\frac{1 + \csc \theta}{\cot \theta + \cos \theta} = \sec \theta$ e. $\cot^3 x + \cot x = \cos x \csc^3 x$ f. $\frac{1}{1 - \cos \theta} + \frac{1}{1 + \cos \theta} = 2 \csc^2 \theta$

4. Find all solution of each equation on the interval $[0, 2\pi]$

- a. $2 \cos x = \sqrt{2}$ b. $\sec^2 x = \frac{4}{3}$ $3 \sin x + 3 = \cos^2 x$

5. Solve each equation for all values of x on the interval $[0, 2\pi]$.

- a. $5 \cos x = 3 \cos x + \sqrt{3}$ b. $\tan^2 x - \tan x = 0$ c. $\csc x - 2 \cot x = 0$

6. Find the exact value of each trigonometric expression

- a. $\sin \frac{7\pi}{12}$ b. $\cos 165^\circ$ c. $\tan 255^\circ$ d. $\cos \frac{17\pi}{12}$ e. $\sin 15^\circ$
f. $\sin 20^\circ \cos 40^\circ + \cos 20^\circ \sin 40^\circ$ g. $\frac{\tan \frac{5\pi}{24} + \tan \frac{\pi}{8}}{1 - \tan \frac{5\pi}{24} \tan \frac{\pi}{8}}$

7. Simplify each expression

- a. $\cos 9x \cos 5x - \sin 9x \sin 5x$ b. $\sin 148^\circ \cos 23^\circ - \cos 148^\circ \sin 23^\circ$

8. Verify:

- a. $\cos\left(x - \frac{\pi}{2}\right) = \sin x$ b. $\tan(\theta - 360^\circ) = \tan \theta$

9. Find the solutions on the interval $[0, 2\pi]$

$$\sin\left(x + \frac{\pi}{4}\right) - \sin\left(x - \frac{\pi}{4}\right) = 0$$