Chp5

1. a reciprocal

$\frac{1}{\sin x} = \csc x \quad \frac{1}{\csc x} = \sin x$

$\frac{1}{\cos x} = \sec x \quad \frac{1}{\sec x} = \cos x$

$\frac{1}{\tan x} = \cot x \quad \frac{1}{\cot x} = \tan x$

b. Quotient

$\frac{\sin x}{\cos x} = \tan x \quad \frac{\cos x}{\sin x} = \cot x$

c. Pythagorean

$\sin^2 x + \cos^2 x = 1 \quad \tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

2a. $\csc x \tan x$ to $\sec x$

$\frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} = \frac{1}{\cos x}$ ↑

b. $\cos^2 x \csc x \sec x \rightarrow \cot x$

$\cos^2 x \cdot \frac{1}{\sin x} \cdot \frac{1}{\cos x} = \frac{\cos x}{\sin x} = \cot x$

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c. $\frac{1-\cos^2 x}{\tan x}$ to $\sin x \cos x$

$\frac{\sin^2 x}{\tan x}$ (pythagorean) $\sin^2 x \cdot \frac{\cos x}{\sin x}$

$= \sin x \cos x$

3a $\sec x(\sec x - \cos x) = \tan^2 x$

$\sec^2 x - \left(\frac{1}{\cos x}\right)\cos x =$ distribute

$\sec^2 x - 1 = \tan^2 x$

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b. $\csc^2 x - \cos^2 x \csc^2 x = 1 \quad a - ba = a(1-b)$

$\csc^2 x (1 - \cos^2 x) = 1$

$\csc^2 x (\sin^2 x) =$ pythagorean

$\frac{1}{\sin^2 x} \cdot \sin^2 x = 1$

3c. $(\sec x+1)(\sec x-1)=\tan^2 x$

$\sec^2 x - 1 = \tan^2 x$ distribute / Pythag.

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d. $\dfrac{1+\csc\theta}{\cot\theta+\cos\theta}=\sec\theta$ $\quad \dfrac{1}{\cos\theta}$

$\dfrac{\left(1+\frac{1}{\sin x}\right)}{\left(\frac{\cos x}{\sin x}+\frac{\cos x}{1}\right)}$ change to $\sin x$ & $\cos x$ common denominators

$\dfrac{\left(\frac{\sin x+1}{\sin x}\right)}{\left(\frac{\cos x+\cos x\sin x}{\sin x}\right)} \rightarrow \dfrac{\sin x\,(\sin x+1)}{\cos x+\cos x\sin x}$

$\dfrac{\sin x+1}{\cos x(1+\sin x)}$ factor $\cos x$, cancel terms

$\dfrac{1}{\cos x}=\sec x$

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e. $\cot^3 x+\cot x=\cos x\csc^3 x$ $\quad \dfrac{1}{\sin^3 x}$

$\cot x(\cot^2 x+1)=$ factor

$\cot x(\csc^2 x)=$

$\dfrac{\cos x}{\sin x}\cdot\dfrac{1}{\sin^2 x}=$ $\rightarrow \sin x$ or $\cos x$

$\cos x\cdot\dfrac{1}{\sin^3 x}=\cos x\csc^3 x$

3f. $\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta} = 2\csc^2\theta$

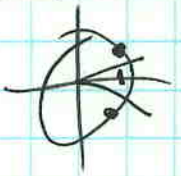
$\frac{1+\cos\theta+1-\cos\theta}{1-\cos^2\theta}$ common denominator multiply

$\frac{2}{\sin^2\theta} = 2\csc^2\theta$

4.a) $2\cos x = \sqrt{2}$

$\cos x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{7\pi}{4}$



b) $\sec^2 x = \frac{4}{3}$

$\frac{1}{\cos^2 x} = \frac{4}{3} \Rightarrow \cos^2 x = \frac{3}{4}$

$\cos x = \frac{\sqrt{3}}{2}$ $x = \frac{\pi}{6}, \frac{11\pi}{6}$

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c) $3\sin x + 3 = \cos^2 x$

$3\sin x + 3 = 1 - \sin^2 x$

$\sin^2 x + 3\sin x + 2 = 0$

$(\sin x + 2)(\sin x + 1) = 0$

$\sin x = -2$ no sol. $\sin x = -1$

$x = \frac{3\pi}{2}$

$x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0$

ZPP

5.a) $5\cos x = 3\cos x + \sqrt{3}$

$2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6} + 2\pi x$

$x = \frac{11\pi}{6} + 2\pi x$

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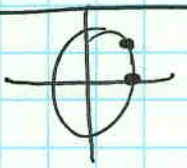
b) $\tan^2 x - \tan x = 0$

$\tan x(\tan x - 1) = 0$

$\tan x = 0$ $\tan x = 1$



$x = 0 + \pi x$

$x = \frac{\pi}{4} + \pi x$

5c. $\csc x - 2\cot x = 0$

$$\frac{1}{\sin x} - \frac{2\cos x}{\sin x} = 0$$

$$\sin x \cdot \frac{1 - 2\cos x}{\sin x} = 0 \cdot \sin x$$

$$1 = 2\cos x$$

$$\frac{1}{2} = \cos x \qquad x = \frac{\pi}{3}, \frac{5\pi}{3}$$

6.a) $\sin\frac{7\pi}{12} = \sin\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right) = \sin\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$

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b) $\cos 165^\circ = \cos(120^\circ + 45^\circ)$

$= \cos 120^\circ \cos 45^\circ - \sin 120^\circ \sin 45^\circ$

$= \frac{-1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{-\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{-\sqrt{2}-\sqrt{6}}{4}$

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c) $\tan 255^\circ = \tan(210^\circ + 45^\circ)$

$\frac{\tan 210^\circ + \tan 45^\circ}{1 - \tan 210^\circ \tan 45^\circ} = \frac{\left(\frac{\sqrt{3}}{3}\right)+1}{1-\frac{\sqrt{3}}{3}\cdot 1} = \frac{\frac{\sqrt{3}+3}{3}}{\frac{3-\sqrt{3}}{3}}$

$= \frac{\sqrt{3}+3}{3-\sqrt{3}}\cdot\frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{3\sqrt{3}+3\sqrt{3}+9+3}{9-3} = \frac{6\sqrt{3}+12}{6} = \sqrt{3}+2$

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d) $\cos\frac{17\pi}{12} = \cos\left(\frac{15\pi}{12}+\frac{2\pi}{12}\right) = \cos\left(\frac{5\pi}{4}+\frac{\pi}{6}\right)$

$\cos\frac{5\pi}{4}\cos\frac{\pi}{6} - \sin\frac{5\pi}{4}\cos\frac{\pi}{6} =$

$\frac{-\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{-\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{-\sqrt{6}+\sqrt{2}}{4}$

6.e) $\sin 15 = \sin\frac{30}{2}$

$= \pm\sqrt{\frac{1-\cos 30}{2}} = \pm\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \pm\sqrt{\frac{\left(\frac{2-\sqrt{3}}{2}\right)}{\left(\frac{2}{1}\right)}}$

$= \pm\sqrt{\frac{2-\sqrt{3}}{4}} = \pm\frac{\sqrt{2-\sqrt{3}}}{2}$

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f) $\sin 20^\circ \cos 40^\circ + \cos 20^\circ \sin 40^\circ = \sin(20+40) =$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

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g) $\frac{\tan\frac{5\pi}{24} + \tan\frac{\pi}{8}}{1 - \tan\frac{5\pi}{24}\tan\frac{\pi}{8}} = \tan\left(\frac{5\pi}{24} + \frac{\pi}{8}\right) = \tan\left(\frac{5\pi}{24} + \frac{3\pi}{24}\right)$

$= \tan\left(\frac{8\pi}{24}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

7.a) $\cos 9x \cos 5x - \sin 9x \sin 5x = \cos(9x+5x) = \cos 14x$

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b) $\sin 148^\circ \cos 23^\circ - \cos 148^\circ \sin 23^\circ = \sin(148^\circ - 23^\circ)$

$= \sin 125^\circ$

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8 a. $\cos\left(x - \frac{\pi}{2}\right) = \sin x$

$\cos x \cos\frac{\pi}{2} + \sin x \sin\frac{\pi}{2} =$

~~$\cos x (0)$~~ $+ \sin x (1)$

$\sin x = \sin x$

b $\tan(\theta - 360^\circ) = \tan\theta$

$\frac{\tan\theta - \tan 360^\circ}{1 + \tan\theta \tan 360^\circ} =$

$\frac{\tan\theta - 0}{1 - \tan\theta(0)} = \tan\theta = \tan\theta$

9. $\sin\left(x+\frac{\pi}{4}\right)-\sin\left(x-\frac{\pi}{4}\right)=0$

$\sin x\cos\frac{\pi}{4}+\cos x\sin\frac{\pi}{4}-\left(\sin x\cos\frac{\pi}{4}-\cos x\sin\frac{\pi}{4}\right)=0$

$\sin x\left(\frac{\sqrt{2}}{2}\right)+\cos x\left(\frac{\sqrt{2}}{2}\right)-\sin x\left(\frac{\sqrt{2}}{2}\right)+\cos x\left(\frac{\sqrt{2}}{2}\right)=0$

$\frac{2\sqrt{2}}{2}\cos x=0$ ← divide by $\frac{2\sqrt{2}}{2}$

$\cos x=0$

$x=\frac{\pi}{2},\ \frac{3\pi}{2}$