Study Guide and Review

Choose the correct term from the list to complete each sentence.
axis of symmetry   center
conic section   conjugate axis
co-vertices   degenerate conic
directrix   eccentricity
ellipse   foci
focus   hyperbola
locus   major axis
minor axis   orientation
parabola   parameter
parametric curve
parametric equation
transverse axis
vertex   vertices

1. A _______ is a figure formed when a plane intersects a double–napped right cone.

SOLUTION:

conic section

ANSWER:

conic section

2. A circle is the _______ of points that fulfill the property that all points be in a given plane and a specified distance from a given point.

SOLUTION:

locus

ANSWER:

locus

3. The _______ of a parabola is perpendicular to its axis of symmetry.

SOLUTION:

directrix

ANSWER:

directrix

4. The co–vertices of a(n) _______ lie on its minor axis, while the vertices lie on its major axis.

SOLUTION:

ellipse

ANSWER:

ellipse

5. From any point on an ellipse, the sum of the distances to the _______ of the ellipse remains constant.

SOLUTION:
foci

ANSWER:
foci

6. The _______ of an ellipse is a ratio that determines how “stretched” or “circular” the ellipse is. It is found using the ratio $\frac{c}{a}$.

SOLUTION:
eccentricity

ANSWER:
eccentricity

7. The _______ of a circle is a single point, and all points on the circle are equidistant from that point.

SOLUTION:
center

ANSWER:
center

8. Like an ellipse, a _______ has vertices and foci, but it also has a pair of asymptotes and does not have a connected graph.

SOLUTION:
hyperbola

ANSWER:
hyperbola

For each equation, identify the vertex, focus, axis of symmetry, and directrix. Then graph the parabola.

11. $(x + 3)^2 = 12(y + 2)$

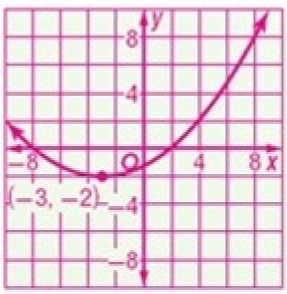
SOLUTION:

$(x + 3)^2 = 12(y + 2)$

The equation is in standard form and the squared term is x, which means that the parabola opens vertically. The equation is in the form $(x - h)^2 = 4p(y - k)$, so $h = -3$ and $k = -2$. Because $4p = 12$ and $p = 3$, the graph opens up. Use the values of h, k, and p to determine the characteristics of the parabola.

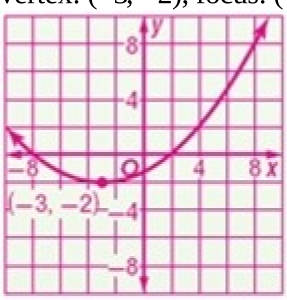
vertex: $(-3, -2)$	(h, k)
directrix: $y = -5$	$y = k - p$
focus: $(-3, 1)$	$(h, k + p)$
axis of symmetry: $x = -3$	$x = h$

Graph the vertex, focus, axis, and directrix of the parabola. Then make a table of values to graph the general shape of the curve.



ANSWER:

vertex: $(-3, -2)$, focus: $(-3, 1)$; axis of symmetry: $x = -3$; directrix: $y = -5$



12. $(y - 2)^2 = 8(x - 5)$

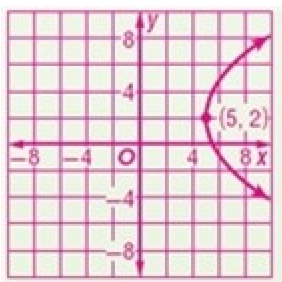
SOLUTION:

$(y - 2)^2 = 8(x - 5)$

The equation is in standard form and the squared term is y, which means that the parabola opens horizontally. The equation is in the form $(y - k)^2 = 4p(x - h)$, so $h = 5$ and $k = 2$. Because $4p = 8$ and $p = 2$, the graph opens to the right. Use the values of h, k, and p to determine the characteristics of the parabola.

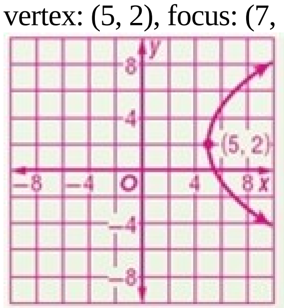
vertex: (5, 2)	(h, k)
directrix: $x = 3$	$x = h - p$
focus: (7, 2)	$(h + p, k)$
axis of symmetry: $y = 2$	$y = k$

Graph the vertex, focus, axis, and directrix of the parabola. Then make a table of values to graph the general shape of the curve.



ANSWER:

vertex: (5, 2), focus: (7, 2); axis of symmetry: $y = 2$; directrix: $x = 3$



13. $(x - 2)^2 = -4(y + 1)$

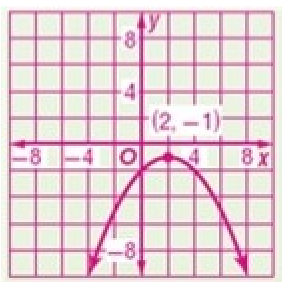
SOLUTION:

$(x - 2)^2 = -4(y + 1)$

The equation is in standard form and the squared term is x, which means that the parabola opens vertically. The equation is in the form $(x - h)^2 = 4p(y - k)$, so $h = 2$ and $k = -1$. Because $4p = -4$ and $p = -1$, the graph opens down. Use the values of h, k, and p to determine the characteristics of the parabola.

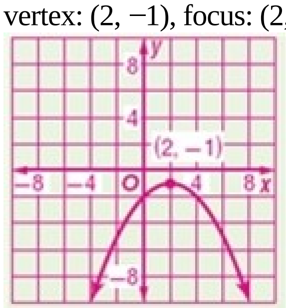
vertex: (2, −1)	(h, k)
directrix: $y = 0$	$y = k - p$
focus: (2, −2)	$(h, k + p)$
axis of symmetry: $x = 2$	$x = h$

Graph the vertex, focus, axis, and directrix of the parabola. Then make a table of values to graph the general shape of the curve.



ANSWER:

vertex: (2, −1), focus: (2, −2); axis of symmetry: $x = 2$; directrix: $y = 0$



14. $(x - 5) = \frac{1}{12}(y - 3)^2$

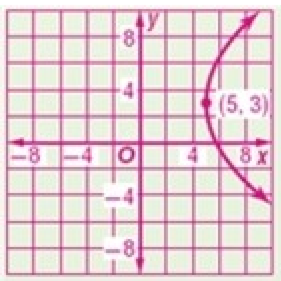
SOLUTION:

$(x - 5) = \frac{1}{12}(y - 3)^2$

The equation is in standard form and the squared term is y, which means that the parabola opens horizontally. The equation is in the form $(y - k)^2 = 4p(x - h)$, so $h = 5$ and $k = 3$. Because $4p = \frac{1}{12}$ and $p = \frac{1}{48}$, the graph opens to the right. Use the values of h, k, and p to determine the characteristics of the parabola.

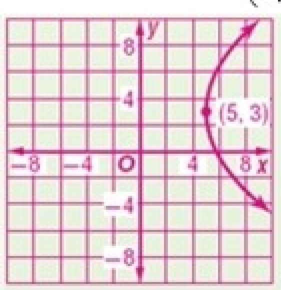
vertex: (5, 3)	(h, k)
directrix: $x = 2\frac{47}{48}$	$x = h - p$
focus: $\left(5\frac{1}{48}, 3\right)$	$(h + p, k)$
axis of symmetry: $y = 3$	$y = k$

Graph the vertex, focus, axis, and directrix of the parabola. Then make a table of values to graph the general shape of the curve.



ANSWER:

vertex: (5, 3), focus: $\left(\frac{241}{48}, 3\right)$; axis of symmetry: $y = 3$; directrix: $x = 2$



Write an equation for and graph a parabola with the given focus F and vertex V.

15. $F(1, 1)$, $V(1, 5)$

SOLUTION:

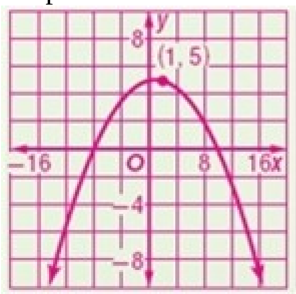
Because the focus and vertex share the same x-coordinate, the graph is vertical. The focus is $(h, k + p)$, so the value of p is $1 - 5$ or -4.

Because p is negative, the graph opens down. Write the equation for the parabola in standard form using the values of h, p, and k.

$$\begin{aligned} 4p(y - k) &= (x - h)^2 \\ 4(-4)(y - 5) &= (x - 1)^2 \\ -16(y - 5) &= (x - 1)^2 \end{aligned}$$

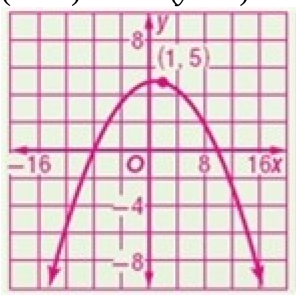
The standard form of the equation is $(x - 1)^2 = -16(y - 5)$.

Graph the vertex and focus. Then make a table of values to graph the parabola.



ANSWER:

$(x - 1)^2 = -16(y - 5)$



16. $F(-3, 6)$, $V(7, 6)$

SOLUTION:

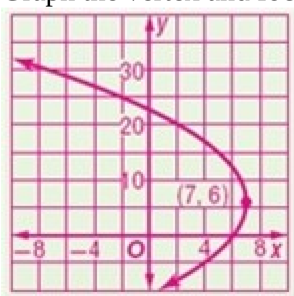
Because the focus and vertex share the same y-coordinate, the graph is horizontal. The focus is $(h + p, k)$, so the value of p is $-3 - 7$ or -10. Because p is negative, the graph opens to the left.

Write the equation for the parabola in standard form using the values of h, p, and k.

$$(y - h)^2 = 4p(x - k)$$
$$(y - 6)^2 = 4(-10)(x - 7)$$
$$(y - 6)^2 = -40(x - 7)$$

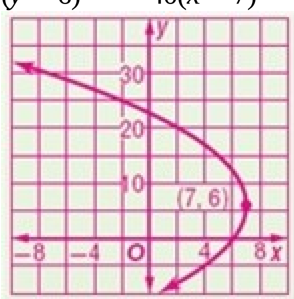
The standard form of the equation is $(y - 6)^2 = -40(x - 7)$.

Graph the vertex and focus. Then make a table of values to graph the parabola.



ANSWER:

$(y - 6)^2 = -40(x - 7)$



17. $F(-2, -3)$, $V(-2, 1)$

SOLUTION:

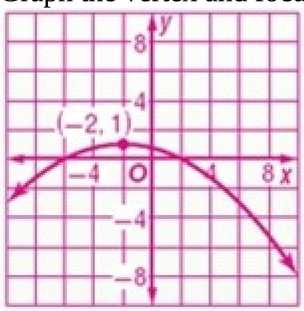
Because the focus and vertex share the same x-coordinate, the graph is vertical. The focus is $(h, k + p)$, so the value of p is $-3 - 1$ or -4. Because p is negative, the graph opens down.

Write the equation for the parabola in standard form using the values of h, p, and k.

$$\begin{aligned} 4p(y-k) &= (x-h)^2 \\ 4(-4)(y-1) &= [x-(-2)]^2 \\ -16(y-1) &= (x+2)^2 \end{aligned}$$

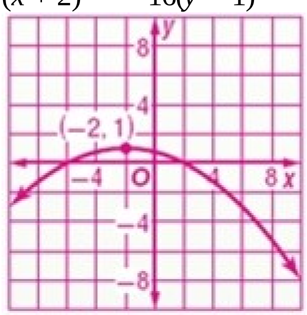
The standard form of the equation is $(x + 2)^2 = -16(y - 1)$.

Graph the vertex and focus. Then make a table of values to graph the parabola.



ANSWER:

$(x + 2)^2 = -16(y - 1)$



18. $F(3, -4)$, $V(3, -2)$

SOLUTION:

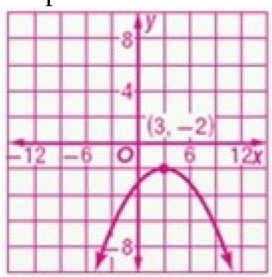
$F(3, -4)$, $V(3, -2)$
Because the focus and vertex share the same x-coordinate, the graph is vertical. The focus is $(h, k + p)$, so the value of p is $-4 - (-2)$ or -2. Because p is negative, the graph opens down.

Write the equation for the parabola in standard form using the values of h, p, and k.

$$\begin{aligned} 4p(y - k) &= (x - h)^2 \\ 4(-2)[y - (-2)] &= (x - 3)^2 \\ -8(y + 2) &= (x - 3)^2 \end{aligned}$$

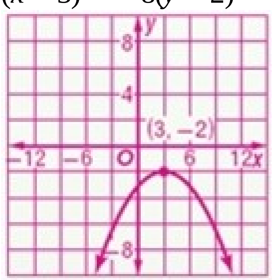
The standard form of the equation is $(x - 3)^2 = -8(y + 2)$.

Graph the vertex and focus. Then make a table of values to graph the parabola.



ANSWER:

$(x - 3)^2 = -8(y + 2)$



Write an equation for and graph each parabola with focus *F* and the given characteristics.

19. *F*(–4, –4); concave left; contains (–7, 0)

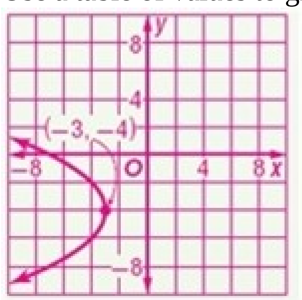
SOLUTION:

Because the parabola opens to the left, the vertex is $(-4 - p, -4)$. Use the standard form of the equation of a horizontal parabola and the point (−7, 0) to find the equation.

$$\begin{aligned} 4p(x - h) &= (y - k)^2 \\ 4p[-7 - (-4 - p)] &= [0 - (-4)]^2 \\ 4p(-3 + p) &= 16 \\ p(-3 + p) &= 4 \\ p^2 - 3p &= 4 \\ p^2 - 3p - 4 &= 0 \\ (p - 4)(p + 1) &= 0 \\ p &= -1 \text{ or } 4 \end{aligned}$$

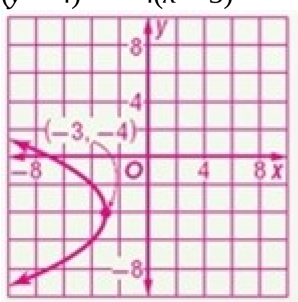
Because the parabola opens to the left, the value of p must be negative. Therefore, $p = -1$. The vertex is (−3, −4) and the standard form of the equation is $(y + 4)^2 = -4(x + 3)$.

Use a table of values to graph the parabola.



ANSWER:

$(y + 4)^2 = -4(x + 3)$



20. $F(-1, 4)$; concave down; contains (7, –2)

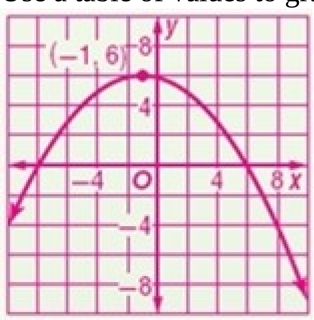
SOLUTION:

Because the parabola opens down, the vertex is $(-1, 4 - p)$. Use the standard form of the equation of a horizontal parabola and the point (7, −2) to find the equation.

$$\begin{aligned} 4p(y-k) &= (x-h)^2 \\ 4p[-2-(4-p)] &= [7-(-1)]^2 \\ 4p(-6+p) &= 64 \\ p(-6+p) &= 16 \\ p^2-6p &= 16 \\ p^2-6p-16 &= 0 \\ (p-8)(p+2) &= 0 \\ p &= 8 \text{ or } -2 \end{aligned}$$

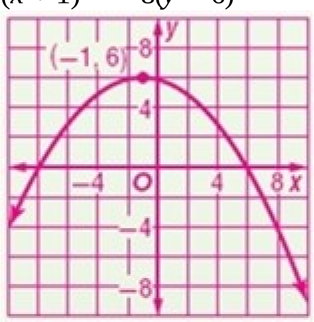
Because the parabola opens down, the value of p must be negative. Therefore, $p = -2$. The vertex is (−1, 6), and the standard form of the equation is $(x + 1)^2 = -8(y - 6)$.

Use a table of values to graph the parabola.



ANSWER:

$(x + 1)^2 = -8(y - 6)$



21. $F(3, -6)$; concave up; contains (9, 2)

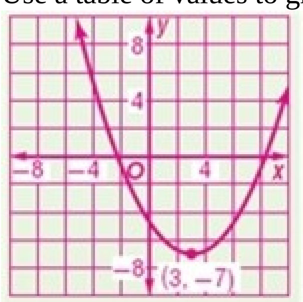
SOLUTION:

Because the parabola opens up, the vertex is $(3, -6 - p)$. Use the standard form of the equation of a horizontal parabola and the point (9, 2) to find the equation.

$$\begin{aligned} 4p(y - k) &= (x - h)^2 \\ 4p[2 - (-6 - p)] &= (9 - 3)^2 \\ 4p(8 + p) &= 36 \\ p(8 + p) &= 9 \\ p^2 + 8p &= 9 \\ p^2 + 8p - 9 &= 0 \\ (p + 9)(p - 1) &= 0 \\ p &= -9 \text{ or } 1 \end{aligned}$$

Because the parabola opens up, the value of p must be positive. Therefore, $p = 1$. The vertex is $(3, -7)$, and the standard form of the equation is $(x - 3)^2 = 4(y + 7)$.

Use a table of values to graph the parabola.



ANSWER:

$(x - 3)^2 = 4(y + 7)$