Graph the hyperbola given by each equation.

30. $\frac{(y+3)^2}{30} - \frac{(x-6)^2}{8} = 1$

SOLUTION:

The equation is in standard form, and $h = 6$ and $k = -3$. Because $a^2 = 30$ and $b^2 = 8$, $a = \sqrt{30} \approx 5.5$ and $b = \sqrt{8}$. The values of a and b can be used to find c.

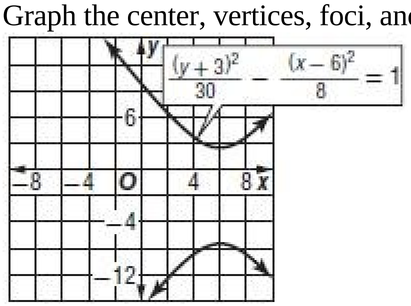
$$c^2 = a^2 + b^2$$
$$c^2 = (\sqrt{30})^2 + (\sqrt{8})^2$$
$$c^2 = 30 + 8$$
$$c = \sqrt{38} \approx 6.2$$

Use h, k, a, b, and c to determine the characteristics of the hyperbola.

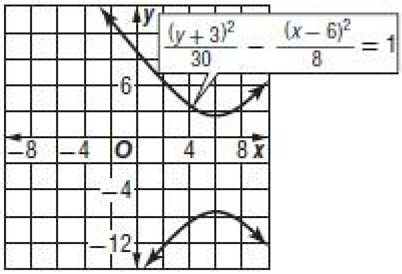
orientation: In the standard form of the equation, the x–term is being subtracted. Therefore, the orientation of the hyperbola is vertical.
center: $(h, k) = (6, -3)$
vertices: $(h, k \pm a) = (6, 2.5)$ and $(6, -8.5)$
foci: $(h, k \pm c) = (6, 3.2)$ and $(6, -9.2)$
asymptotes:

$$y - k = \pm \frac{a}{b}(x - h)$$
$$y - (-3) = \pm \frac{\sqrt{30}}{2\sqrt{2}}(x - 6)$$
$$y + 3 = \pm \frac{\sqrt{15}}{2}(x - 6)$$
$$y + 3 = \pm \left[\frac{\sqrt{15}}{2}x - 3\sqrt{15}\right]$$
$$y = \pm \left[\frac{\sqrt{15}}{2}x - 3\sqrt{15}\right] - 3$$
$$y = \frac{\sqrt{15}}{2}x - 3\sqrt{15} - 3 \quad OR$$
$$y = -\frac{\sqrt{15}}{2}x + 3\sqrt{15} - 3$$

Graph the center, vertices, foci, and asymptotes. Then make a table of values to sketch the hyperbola.



ANSWER:



31. $\frac{(x+7)^2}{18} - \frac{(y-6)^2}{36} = 1$

SOLUTION:

$$\frac{(x+7)^2}{18} - \frac{(y-6)^2}{36} = 1$$

The equation is in standard form, and $h = -7$ and $k = 6$. Because $a^2 = 18$ and $b^2 = 36$, $a = \sqrt{18} \approx 4.2$ and $b = 6$. The values of a and b can be used to find c.

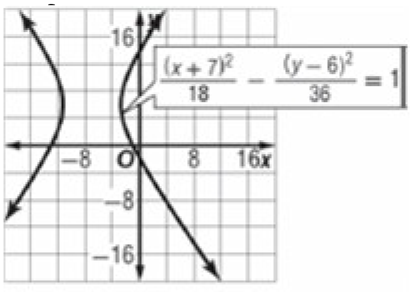
$$c^2 = a^2 + b^2$$
$$c^2 = 18 + 36$$
$$c^2 = 54$$
$$c = \sqrt{54} \approx 7.3$$

Use h, k, a, b, and c to determine the characteristics of the hyperbola.

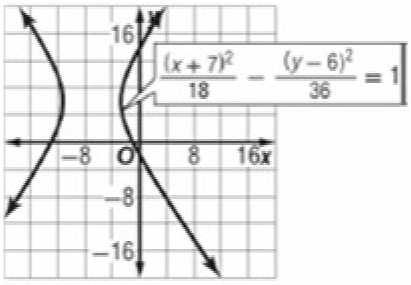
orientation: In the standard form of the equation, the y−term is being subtracted. Therefore, the orientation of the hyperbola is horizontal.
center: $(h, k) = (-7, 6)$
vertices: $(h \pm a, k) = (-11.2, 6)$ and $(-2.8, 6)$
foci: $(h \pm c, k) = (0.3, 6)$ and $(-14.3, 6)$
asymptotes:

$$y - k = \pm\frac{b}{a}(x - h)$$
$$y - 6 = \pm\frac{6}{3\sqrt{2}}(x - [-7])$$
$$y - 6 = \pm\sqrt{2}(x + 7)$$
$$y - 6 = \pm\left[\sqrt{2}x + 7\sqrt{2}\right]$$
$$y = \pm\left[\sqrt{2}x + 7\sqrt{2}\right] + 6$$
$$y = \left[\sqrt{2}x + 7\sqrt{2} + 6 \quad OR\right.$$
$$y = -\left[\sqrt{2}x - 7\sqrt{2} + 6\right.$$

Graph the center, vertices, foci, and asymptotes. Then make a table of values to sketch the hyperbola.



ANSWER:



32. $\frac{(y-1)^2}{4} - (x+1)^2 = 1$

SOLUTION:

$$\frac{(y-1)^2}{4} - (x+1)^2 = 1$$

The equation is in standard form, and $h = -1$ and $k = 1$. Because $a^2 = 4$ and $b^2 = 1$, $a = 2$ and $b = 1$. The values of a and b can be used to find c.

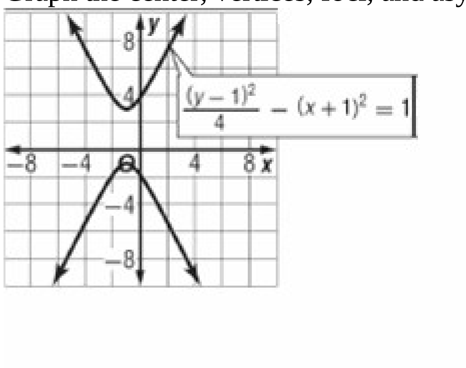
$$c^2 = a^2 + b^2$$
$$c^2 = 4 + 1$$
$$c^2 = 5$$
$$c = \sqrt{5} \approx 2.2$$

Use h, k, a, b, and c to determine the characteristics of the hyperbola.

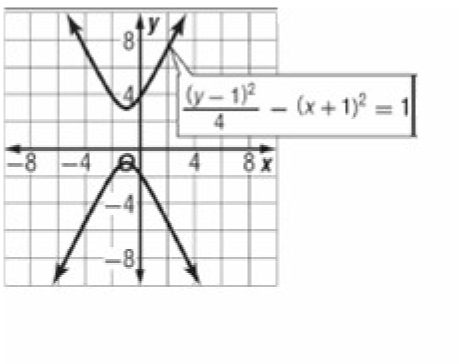
orientation: In the standard form of the equation, the x–term is being subtracted. Therefore, the orientation of the hyperbola is vertical.
center: $(h, k) = (-1, 1)$
vertices: $(h, k \pm a) = (-1, 3)$ and $(-1, -1)$
foci: $(h, k \pm c) = (-1, 3.2)$ and $(-1, -1.2)$
asymptotes:

$$
\begin{aligned}
y - k &= \pm\frac{a}{b}(x - h)\\
y - 1 &= \pm\frac{2}{1}(x - [-1])\\
y - 1 &= \pm 2(x + 1)\\
y - 1 &= \pm[2x + 2]\\
y &= \pm[2x + 2] + 1\\
y &= 2x + 3 \quad OR\\
y &= -2x - 1
\end{aligned}
$$

Graph the center, vertices, foci, and asymptotes. Then make a table of values to sketch the hyperbola.



ANSWER:



33. $x^2 - y^2 - 2x + 4y - 7 = 0$

SOLUTION:

Convert the equation to standard form.

$$
\begin{aligned}
x^2 - y^2 - 2x + 4y - 7 &= 0\\
x^2 - 2x + 1 - (y^2 - 4y + 4) - 7 - 1 + 4 &= 0\\
(x - 1)^2 - (y - 2)^2 &= 4\\
\frac{(x - 1)^2}{4} - \frac{(y - 2)^2}{4} &= 1
\end{aligned}
$$

The equation is in standard form, and $h = 1$ and $k = 2$. Because $a^2 = 4$ and $b^2 = 4$, $a = 2$ and $b = 2$. The values of a and b can be used to find c.

$$c^2 = a^2 + b^2$$

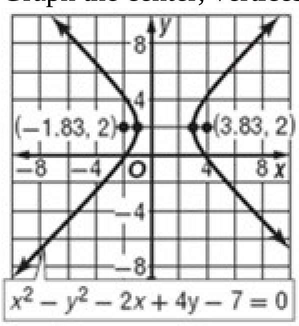
$$c^2 = 4 + 4$$
$$c^2 = 8$$
$$c = \sqrt{8} \approx 2.8$$

Use h, k, a, b, and c to determine the characteristics of the hyperbola.

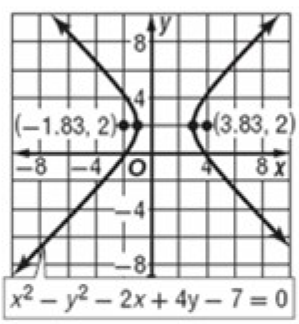
orientation: In the standard form of the equation, the y−term is being subtracted. Therefore, the orientation of the hyperbola is horizontal.
center: $(h, k) = (1, 2)$
vertices: $(h \pm a, k) = (-1, 2)$ and $(3, 2)$
foci: $(h \pm c, k) = (-1.8, 2)$ and $(3.8, 2)$
asymptotes:

$$y - k = \pm\frac{b}{a}(x - h)$$
$$y - 2 = \pm\frac{2}{2}(x - 1)$$
$$y - 2 = \pm(x - 1)$$
$$y = \pm[x + 1] + 2$$
$$y = x + 3 \quad OR$$
$$y = -x + 1$$

Graph the center, vertices, foci, and asymptotes. Then make a table of values to sketch the hyperbola.



ANSWER:



Write an equation for the hyperbola with the given characteristics.

34. vertices (7, 0), (−7, 0); conjugate axis length of 8

SOLUTION:

Because the y−coordinates of the vertices are the same, the transverse axis is horizontal, and the standard form of the equation is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$.

The center is the midpoint of the segment between the vertices, or (0, 0). So, $h = 0$ and $k = 0$. The length of the conjugate axis of a hyperbola is $2b$. So, $2b = 8$, $b = 4$, and $b^2 = 16$. You can find a by determining the distance from a vertex to the center. One vertex is located at (7, 0), which is 7 units from (0, 0). So, $a = 7$.

Using the values of h, k, a, and b, the equation for the hyperbola is $\frac{x^2}{49} - \frac{y^2}{16} = 1$.

ANSWER:

$\frac{x^2}{49} - \frac{y^2}{16} = 1$

35. foci (0, 5), (0, −5); vertices (0, 3), (0, −3)

SOLUTION:

Because the y−coordinates of the vertices are the same, the transverse axis is horizontal, and the standard form of the equation is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$.

The center is the midpoint of the segment between the foci, or (0, 0). So, $h = 0$ and $k = 0$. You can find c by determining the distance from a focus to the center. One focus is located at (0, 5), which is 5 units from (0, 0). So, $c = 5$ and $c^2 = 25$. You can find a by determining the distance from a vertex to the center. One vertex is located at (0, 3), which is 3 units from (0, 0). So, $a = 3$ and $a^2 = 9$.

Now you can use the values of c and a to find b.

$$\begin{aligned} b^2 &= c^2 - a^2 \\ b^2 &= 25 - 9 \\ b^2 &= 16 \\ b &= 4 \end{aligned}$$

Using the values of h, k, a, and b, the equation for the hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$.

ANSWER:

$\frac{y^2}{9} - \frac{x^2}{16} = 1$

36. foci (1, 15), (1, −5); transverse axis length of 16

SOLUTION:

Because the x−coordinates of the foci are the same, the transverse axis is vertical, and the standard form of the equation is $\frac{(y-h)^2}{a^2} - \frac{(x-k)^2}{b^2} = 1$.

The center is the midpoint of the segment between the foci, or (1, 5). So, $h = -1$ and $k = -5$. You can find c by determining the distance from a focus to the center. One focus is located at (1, 15), which is 10 units from (1, 5). So, $c = 10$ and $c^2 = 100$. The length of the transverse axis is 16, so $a = 8$ and $a^2 = 64$.

Now you can use the values of c and a to find b.

$$\begin{aligned} b^2 &= c^2 - a^2 \\ b^2 &= 100 - 64 \\ b^2 &= 36 \\ b &= 6 \end{aligned}$$

Using the values of h, k, a, and b, the equation for the hyperbola is $\frac{(y-5)^2}{64} - \frac{(x-1)^2}{36} = 1$.

ANSWER:

$$\frac{(y-5)^2}{64} - \frac{(x-1)^2}{36} = 1$$

37. vertices (2, 0), (−2, 0); asymptotes $y = \pm\frac{3}{2}x$

SOLUTION:

Because the y−coordinates of the foci are the same, the transverse axis is horizontal, and the standard form of the equation is $\frac{(x-k)^2}{a^2} - \frac{(y-h)^2}{b^2} = 1$.

The center is the midpoint of the segment between the vertices, or (0, 0).So, $h = 0$ and $k = 0$. To find a, determine the distance from a vertex to the center. One vertex is located at (2, 0), which is 2 units from (0, 0). So, $a = 2$ and $a^2 = 4$. Because the slopes of the asymptotes are $\pm\frac{b}{a}$, using the positive slope $\frac{3}{2}$, $b = 3$ and $b^2 = 9$.

Using the values of h, k, a, and b, the equation for the hyperbola is $\frac{x^2}{4} - \frac{y^2}{9} = 1$.

ANSWER:

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

Use the discriminant to identify each conic section.

38. $x^2 - 4y^2 - 6x - 16y - 11 = 0$

SOLUTION:

$$\begin{aligned} B^2 - 4AC &= 0 - 4(1)(-4) \\ &= 16 \end{aligned}$$

The discriminant is greater than 0, so the conic is a hyperbola.

ANSWER:

hyperbola

39. $4y^2 - x - 40y + 107 = 0$

SOLUTION:

$$\begin{aligned} B^2 - 4AC &= 0 - 0(4) \\ &= 0 \end{aligned}$$

The discriminant is 0, so the conic is a parabola.

ANSWER:

parabola

40. $9x^2 + 4y^2 + 162x + 8y + 732 = 0$

SOLUTION:

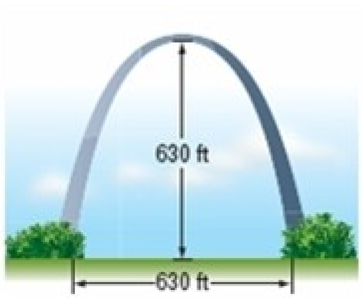
$$\begin{aligned} B^2 - 4AC &= 0 - 4(9)(4) \\ &= -144 \end{aligned}$$

The discriminant is less than 0 and $A \neq C$, so the conic is an ellipse.

ANSWER:

ellipse

56. **MONUMENTS** The St. Louis Arch is in the shape of a *catenary*, which resembles a parabola.
a. Write an equation for a parabola that would approximate the shape of the arch.
b. Find the location of the focus of this parabola.



SOLUTION:

a. With the vertex at the origin, the parabola goes through (0, 0), (315, −630), and (−315, −630). Use a system of equations to determine a, b, and c in $y = ax^2 + bx + c$.

$0a + 0b + 0c = 0$
$(315)^2a + 315b + 0c = -630$
$(-315)^2a - 315b + 0c = -630$

$c = 0$

$$\begin{aligned} 99{,}225a + 315b &= -630 \\ \underline{+\,99{,}225a - 315b} &= \underline{-630} \\ 198{,}450a &= -1260 \\ a &= \frac{2}{315} \end{aligned}$$

Substitute and solve for b.

$$\begin{aligned} 99{,}225 \cdot \frac{2}{315} + 315b &= -630 \\ 630b &= -630 \\ b &= -1 \end{aligned}$$

The equation is $y = -\frac{2}{315}x^2 + 630$.

b. Convert the equation to standard form.

$$\begin{aligned} y &= -\frac{2}{315}x^2 + 630 \\ y - 630 &= -\frac{2}{315}x^2 \\ -\frac{315}{2}(y - 630) &= x^2 \end{aligned}$$

The value of p is $-\frac{315}{2} \div 4$ or -39.375. The focus of the parabola is $630 - 39.375$ or 590.625 feet above the ground.

ANSWER:

a. Sample answer: $y = -\frac{2}{315}x^2 + 630$

b. 590.625 ft above the ground.

58. **ENERGY** Cooling towers at a power plant are in the shape of a hyperboloid. The cross section of a hyperboloid is a hyperbola.

a. Write an equation for the cross section of a tower that is 50 feet tall and 30 feet wide.

b. If the ratio of the height to the width of the tower increases, how is the equation affected?

SOLUTION:

a. The height of the cross section is 50 feet, which is $2a$. So, $a = 25$ and $a^2 = 625$. The width of the cross section is 30 feet, which is $2b$. So, $b = 15$ and $b^2 = 225$.

The transverse axis is vertical, so the equation for the cross section is $\frac{x^2}{225} - \frac{y^2}{625} = 1$.

b. Sample answer: The ratio of the denominator associated with y to the denominator associated with x will increase.

ANSWER:

a. $\frac{x^2}{225} - \frac{y^2}{625} = 1$

b. Sample answer: The ratio of the denominator associated with y to the denominator associated with x will increase.