

7-3 Study Guide and Intervention

(continued)

Hyperbolas

Identify Conic Sections You can determine the type of conic when the equation for the conic is in general form, $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. The discriminant, or $B^2 - 4AC$, can be used to identify a conic when the equation is in general form.

Discriminant	Conic Section
less than 0; $B = 0$ and $A = C$	circle
less than 0; $B \neq 0$ or $A \neq C$	ellipse
equal to 0	parabola
greater than 0	hyperbola

Exercises

Use the discriminant to identify each conic section.

1. $4x^2 + 4y^2 - 2x - 9y + 1 = 0$

$$0^2 - 4(4)(4) < 0 \text{ circle}$$

2. $10x^2 + 6y^2 - x + 8y + 1 = 0$

$$0 - 4(10)(6) < 0 \text{ ellipse}$$

3. $-2x^2 + 6xy + y^2 - 4x - 5y + 2 = 0$

$$6^2 - 4(-2)(1) = 36 + 8 > 0 \text{ hyp.}$$

4. $x^2 + 6xy + y^2 - 2x + 1 = 0$

$$6^2 - 4(1)(1) > 0 \text{ hyperbola}$$

5. $5x^2 + 2xy + 4y^2 + x + 2y + 17 = 0$

$$2^2 - 4(5)(4) < 0 \text{ ellipse}$$

6. $x^2 + 2xy + y^2 + x + 10 = 0$

$$2^2 - 4(1)(1) = 0 \text{ parabola}$$

7. $25x^2 + 100x - 54y = -200$

$$0^2 - 4(25)(-54) = 0 \text{ parabola}$$

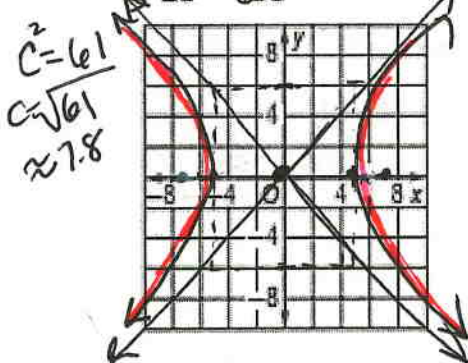
8. $16x^2 + 100x - 54y^2 = -100$

$$0 - 4(16)(-54) > 0 \text{ hyperbola}$$

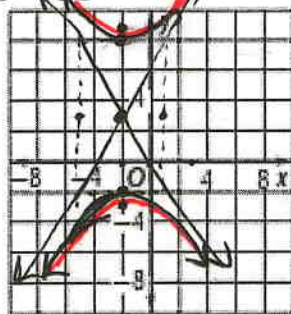
Exercises

Graph the hyperbola given by each equation.

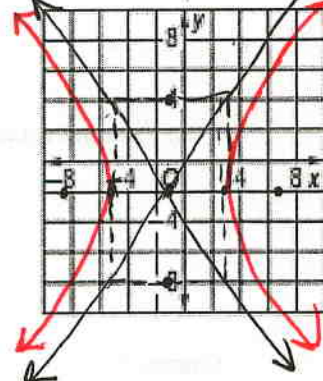
1. $\frac{x^2}{25} - \frac{y^2}{36} = 1$



2. $\frac{(y-3)^2}{25} - \frac{(x+2)^2}{9} = 1$



3. $\frac{(x-1)^2}{16} - \frac{(y+2)^2}{36} = 1$



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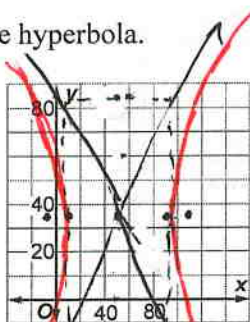
Word Problem Practice

Hyperbolas

1. **EARTHQUAKES** The epicenter of an earthquake lies on a branch of the hyperbola represented by $\frac{(x-50)^2}{1600} - \frac{(y-35)^2}{2500} = 1$, where the seismographs are located at the foci.

a. Graph the hyperbola.

$a=40$
 $b=50$
 $c \approx 64$
 $O(50, 35)$
 114
 -14

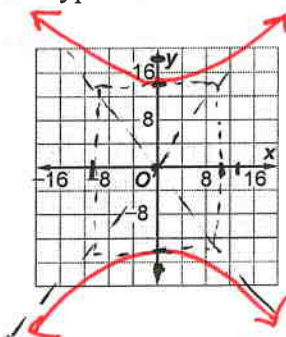


b. Find the locations of the seismographs.
 seismograph locations:
 $(-14, 35)$ & $(114, 35)$

2. **SHADOWS** A lamp projects light onto a wall in the shape of a hyperbola. The edge of the light can be modeled by $\frac{y^2}{196} - \frac{x^2}{121} = 1$.

a. Graph the hyperbola.

$a=14$
 $b=11$
 $c=17.6$



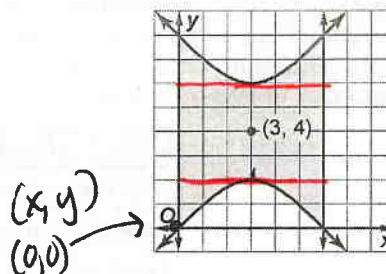
b. Write the equations of the asymptotes.

$y = \pm \frac{14}{11}x$

c. Find the eccentricity.

$e = \frac{c}{a} = \frac{17.6}{14} = 1.27$

3. **PARKS** A grassy play area is in the shape of a hyperbola, as shown.



$h=3$
 $k=4$
 $a=2$
 $b^2=9$
 $x=0$
 $y=0$
 $b=3$

a. Write an equation that models the curved sides of the play area.

$\frac{(y-4)^2}{4} - \frac{(x-3)^2}{9} = 1$
 $\frac{(0-4)^2}{4} - \frac{(-3)^2}{9} = 1$
 $4 - \frac{9}{9} = 1$
 $3 = \frac{9}{b^2}$
 $b^2 = 3$
 $b = \sqrt{3}$

b. If each unit on the coordinate plane represents 3 feet, what is the narrowest vertical width of the play area?

2a - narrowest point
 4 units
 12 feet

4. Use the discriminant to identify each conic section.

- a. $-2x^2 + 6xy + y^2 - 4x - 5y + 2 = 0$
 $b^2 - 4ac = (-2)(1) - 4(3)(1) = -14 < 0$ hyperbola
 b. $x^2 + 6xy + y^2 - 2x + 1 = 0$
 $b^2 - 4ac = (6)^2 - 4(1)(1) = 32 > 0$ hyperbola
 c. $5x^2 + 2xy + 4y^2 + x + 2y + 17 = 0$
 $b^2 - 4ac = (2)^2 - 4(5)(4) = -76 < 0$ ellipse
 d. $x^2 + 2xy + y^2 + x + 10 = 0$
 $b^2 - 4ac = (2)^2 - 4(1)(1) = 0$ parabola