7-3-2 Hyperbolic Equations

Ex 2 General Form to Standard form

$25x^2 - 16y^2 + 100x + 96y = 444$ * subtracting square terms

$25x^2 + 100x - 16y^2 + 96y = 444$

$25(x^2 + 4x + 4 - 4) - 16(y^2 - 6y + 9 - 9) = 444$

$25((x+2)^2 - 4) - 16((y-3)^2 - 9) = 444$

$25(x+2)^2 - 100 - 16(y-3)^2 + 144 = 444$ $= 400$

$+100$ -144 -44

$$\frac{25(x+2)^2}{400} - \frac{16(y-3)^2}{400} = 1 \Rightarrow \boxed{\frac{(x+2)^2}{16} - \frac{(y-3)^2}{25} = 1}$$

You try: $2x^2 - 3y^2 + 8x - 12y - 28 = 0$

$2(x^2 + 4x + 4 - 4) - 3(y^2 + 4y + 4 - 4) = 28$

$2(x+2)^2 - 8 - 3(y+2)^2 + 12 = 28$

$2(x+2)^2 - 3(y+2)^2 = 24$

$$\frac{(x+2)^2}{12} - \frac{(y+2)^2}{8} = 1$$

Writing equations given Hyperbolic Characteristics

horz $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ vert: $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

need: h, k, a, b

transitive axis: 2a, a, midpoint center h, k
Vertices orientation

conjugate axis: 2b, b, midpoint center h, k, orientation

focal axis, foci: 2c, c, midpoint center h, k, orientation

asymptote equation: gives slope: $\frac{\text{rise}}{\text{run}}$ $\frac{b}{a}$ (h) or $\frac{a}{b}$ (v)

$c^2 = a^2 + b^2$ $e = \frac{c}{a}$

Ex 3 Write a hyperbolic Equation

given Vertices (-3, 0) (-9, 0) ... b | a

asymptote $y = 2x - 12$

TA: $2a = 6$ Center (-6, 0) Slope $= 2 = \frac{b}{a} = \frac{b}{3}$
$a = 3$ $b = 6$

$$\frac{(x+6)^2}{9} - \frac{(y)^2}{36} = 1$$