# 7.2 Ellipses and Circles Notes

## ellipse

*center*

(h,k)

*co-vertex*

on minor axis

*ellipse*

*focus*

defines ellipse

c from center

*major axis*

Long

a from center

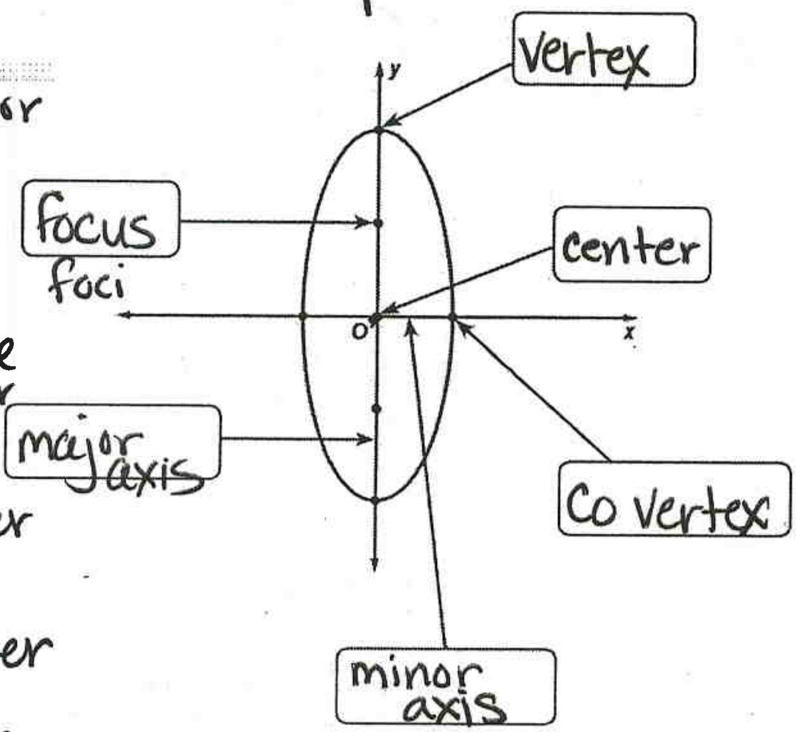
*minor axis*

Short

b from center

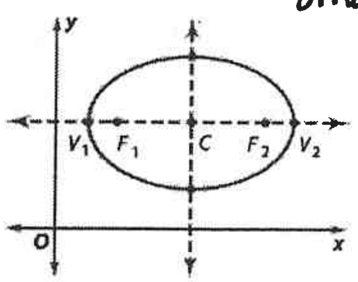
*vertex*

on major axis




**KeyConcept** **Standard Forms of Equations for Ellipses**

big → $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ ← small




**Orientation:** horizontal major axis

**Center:** $(h, k)$ x over a

**Foci:** $(h \pm c, k)$

**Vertices:** $(h \pm a, k)$

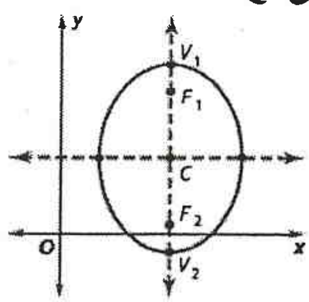
**Co-vertices:** $(h, k \pm b)$

**Major axis:** $y = k$

**Minor axis:** $x = h$

**a, b, c relationship:** $c^2 = a^2 - b^2$ or $c = \sqrt{a^2 - b^2}$

big → $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$ ← small




**Orientation:** vertical major axis

**Center:** $(h, k)$ y over a

**Foci:** $(h, k \pm c)$

**Vertices:** $(h, k \pm a)$

**Co-vertices:** $(h \pm b, k)$

**Major axis:** $x = h$

**Minor axis:** $y = k$

**a, b, c relationship:** $c^2 = a^2 - b^2$ or $c = \sqrt{a^2 - b^2}$

# 7-2-1 Ellipses

equation $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$   ← OR →

center $(h,k)$

center to vertex $= a$
center to co-vertex $= b$
center to focus $= c$

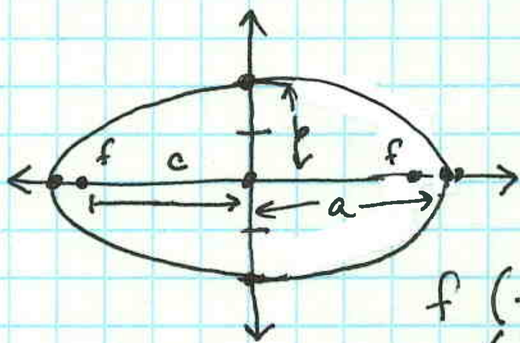
$c = \sqrt{a^2 - b^2}$

$e = \dfrac{c}{a}$

## Graphing Ellipses

$\dfrac{x^2}{16} + \dfrac{y^2}{4} = 1$

$a^2 \rightarrow 16$    $4 \leftarrow b^2$



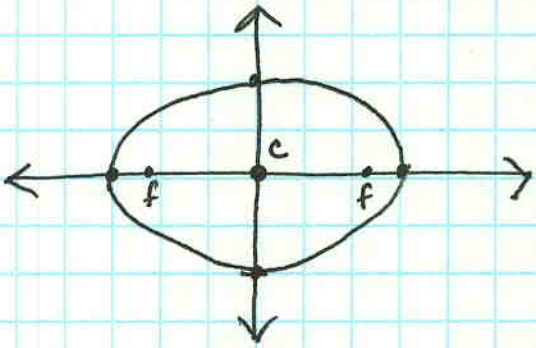

$c = (0,0)$
$a = 4$
$b = 2$
$c = \sqrt{12} \approx 3.5$

f $(-3.5, 0)$
$(3.5, 0)$

$4x^2 + 9y^2 = 36$    not standard form

$\dfrac{4x^2}{36} + \dfrac{9y^2}{36} = \dfrac{36}{36}$

$\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$




center $(0,0)$
$a = 3$
$b = 2$
$c = \sqrt{5}$
$\approx 2.25$

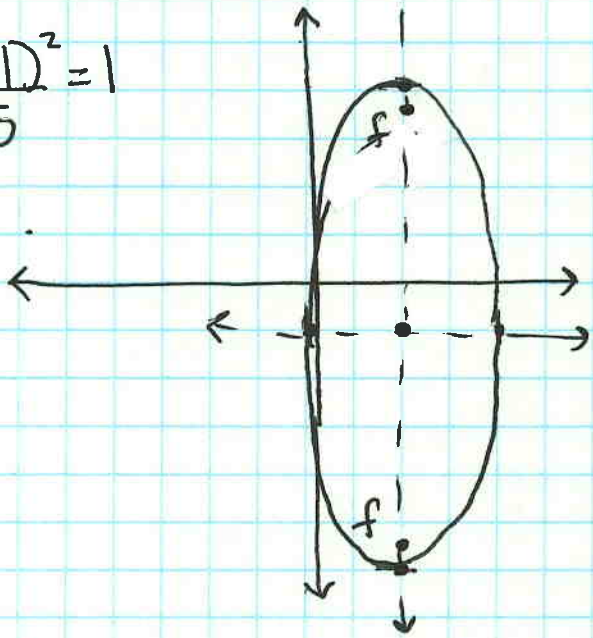
f $(-2.25, 0)$
$(2.25, 0)$

$$\frac{(x-2)^2}{4}+\frac{(y+1)^2}{25}=1$$

center $(2,-1)$

$a=5$
$b=2$
$c=\sqrt{21}=4.6$

$f=(2,-5.6)$
$(2,\ 3.6)$




## Finding General Form

$9x^2+25y^2-36x+50y-164=0$ * general form

$9x^2-36x\ \ +25y^2+50y\ \ =164$ * group x's & y's

$9(x^2-4x\quad)+25(y^2+2y\quad)=164$ * factor to get a=1

$9(x^2-4x+4-4)+25(y^2+2y+1-1)=164$ * +/− $\left(\frac{b}{2}\right)^2$ inside each set

$9((x-2)^2-4)+25((y+1)^2-1)=164$ * complete squares

$9(x-2)^2-36+25(y+1)^2-25=164$ * distribute

$\to +61$

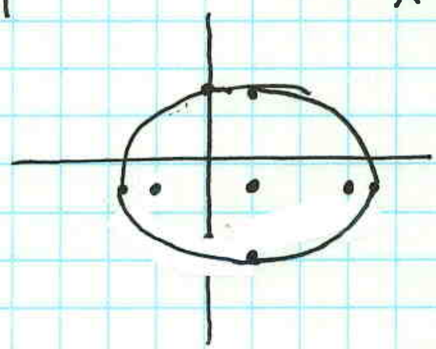
$9(x-2)^2+25(y+1)^2=225$ * combine c's

$\frac{9(x-2)^2}{225}+\frac{25(y+1)^2}{225}=1$ * divide by 225 to get =1

$\frac{(x-2)^2}{25}+\frac{(y+1)^2}{9}=1$ * reduce

graph: center $(2,-1)$
$a=5$
$b=3$
$c=\sqrt{16}$
$=4$

$f=(6,-1)$
$(-2,-1)$



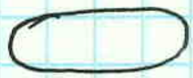
# Eccentricity of Ellipse

How different the ellipse is from a circle.

ratio $\dfrac{\text{focus length}}{\text{major axis}}$ $\qquad e = \dfrac{c}{a}$

$0 < \text{eccentricity} < 1$

more circular $\qquad\qquad$ more elongated



## Ex 3 Eccentricity

$$\frac{(x-4)^2}{13} + \frac{(y+7)^2}{4} = 1$$ standard form

$e = \dfrac{c}{a}$ $\qquad a^2 = 13 \quad b^2 = 4 \qquad c^2 = a^2 - b^2$

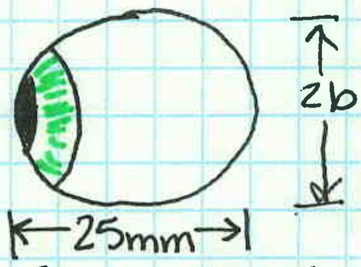
$a = \sqrt{13} \approx 3.6 \qquad c^2 = 13 - 4 = 9$

$c = 3$

$e = \dfrac{3}{3.6} \approx \underline{.832}$ elongated

## Ex 4. Using Eccentricity

$e = .28$ find height




$2a \quad \underline{a = 12.5}$

$e = \dfrac{c}{a} \qquad .28 = \dfrac{c}{12.5} \qquad \underline{c = 3.5}$

$c^2 = a^2 - b^2$

$3.5^2 = 12.5^2 - b^2$

$\boxed{\text{height} = 24\text{mm}}$

$b^2 = 12.5^2 - 3.5^2$

$b^2 = 144 \qquad b = 12$

438: 2-4, 15-18, 44-46