6-2-2 Inverses & Determinant

Recall: multiplicative Inverse:

the MI of a: $\frac{1}{a}$ or a^{-1}

$a \cdot \frac{1}{a} = 1$ $\quad$ $a\,a^{-1} = 1$ $\qquad (A^{-1})AX = B(A^{-1})$

$X = (A^{-1})B$

Inverse Matrix → square matrix

A^{-1} = inverse of A

$A^{-1}A = AA^{-1} = I$ ← identity Matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Square Matrix $N \times N$

1's on the diagonal

0's everywhere else

Verify an Inverse

verify A & B are inverses. Find AB, BA

$$A = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} (-3+4) & (6-6) \\ (-2+2) & (4-3) \end{bmatrix} \qquad BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\updownarrow$

$$\begin{bmatrix} (-3+4) & (2-2) \\ (-6+6) & (4-3) \end{bmatrix}$$

A & B are inverses

$B = A^{-1}$ $\quad$ $A = B^{-1}$

Determinant

determines if a Matrix has an inverse.

Singular Matrix → no inverse

Non-singular Matrix → has an inverse

If determinant = 0, then the Matrix is singular

Finding the determinant: (2×2)

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \det(A) = \underset{①}{ad} - \underset{②}{bc}$$

② ①

Find the inverse of A ($\det(A) \neq 0$)

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & \ominus b \\ \ominus c & a \end{bmatrix}$$

Find det(A) and A^{-1} (if it exists)

$$A = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix} \qquad \det(A) = 8 - (-12) = \underline{\underline{20}}$$

-12 $\quad$ 8

A^{-1} exists.

$$\frac{1}{\det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{20}\begin{bmatrix} 4 & +3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{20} & \frac{3}{20} \\ -\frac{4}{20} & \frac{2}{20} \end{bmatrix} = \underline{\underline{\begin{bmatrix} \frac{1}{5} & \frac{3}{20} \\ -\frac{1}{5} & \frac{1}{10} \end{bmatrix}}}$$

$AA^{-1} = I$

$$\begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} 4/20 & 3/20 \\ -4/20 & 2/20 \end{bmatrix} = \begin{bmatrix} \left(\frac{8}{20}+\frac{12}{20}\right) & \left(\frac{6}{20}-\frac{6}{20}\right) \\ \left(\frac{16}{20}-\frac{16}{20}\right) & \left(\frac{12}{20}+\frac{8}{20}\right) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

HW 383: 24, 25, 29, 30, 36–38