

6-2-2 INVERSES & Determinant

Recall: multiplicative Inverse:

the MI of a : $\frac{1}{a}$ or a^{-1}

$$a \cdot \frac{1}{a} = 1$$

$$a a^{-1} = 1$$

$$(A^{-1})AX = \overbrace{B(A^{-1})}$$

$$X = (A^{-1})B$$

Inverse Matrix \rightarrow square matrix

A^{-1} = inverse of A

$$A^{-1}A = A A^{-1} = I \leftarrow \text{identity Matrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Square Matrix $N \times N$

1's on the diagonal
0's everywhere else

Verify an Inverse

Verify A & B are inverses. Find AB, BA

$$A = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} (-3+4) & (6-6) \\ (-2+2) & (4-3) \end{bmatrix} \quad BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{array}{l} A \& B \\ \text{are inverses} \\ B = A^{-1} \quad A = B^{-1} \end{array}$$

$$\begin{bmatrix} (-3+4) & (2-2) \\ (-6+6) & (4-3) \end{bmatrix}$$

Determinant

determines if a Matrix has an inverse.

Singular Matrix \rightarrow no inverse

Non-singular Matrix \rightarrow has an inverse

If determinant = 0, then the Matrix is singular

Finding the determinant: (2×2)

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \det(A) = ad - bc$$

Diagram illustrating the calculation of the determinant for a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. The matrix is shown with a purple 'X' connecting the top-left element 'a' to the bottom-right element 'd' (labeled with a circled 1) and the top-right element 'b' to the bottom-left element 'c' (labeled with a circled 2). The determinant formula $\det(A) = ad - bc$ is written to the right, with the '1' in 'ad' and the '2' in 'bc' circled in purple to correspond to the paths in the diagram.

Find the inverse of A ($\det(A) \neq 0$)

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Find $\det(A)$ and A^{-1} (if it exists)

$$A = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix} \quad \det(A) = 8 - (-12) = \underline{\underline{20}}$$

$\begin{matrix} \swarrow & \searrow \\ -12 & 8 \end{matrix}$

$\frac{1}{\det} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

A^{-1} exists.

$$A^{-1} = \frac{1}{20} \begin{bmatrix} 4 & 13 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{20} & \frac{3}{20} \\ -\frac{4}{20} & \frac{2}{20} \end{bmatrix} = \underline{\underline{\begin{bmatrix} \frac{1}{5} & \frac{3}{20} \\ -\frac{1}{5} & \frac{1}{10} \end{bmatrix}}}$$

$$AA^{-1} = I$$

$$\begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} \frac{4}{20} & \frac{3}{20} \\ -\frac{4}{20} & \frac{2}{20} \end{bmatrix} = \begin{bmatrix} \left(\frac{8}{20} + \frac{12}{20}\right) & \left(\frac{6}{20} - \frac{6}{20}\right) \\ \left(\frac{16}{20} - \frac{16}{20}\right) & \left(\frac{12}{20} + \frac{8}{20}\right) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

HW 383: 24, 25, 29, 30, 36-38