# 6-2 Matrix Multiplication, Inverses and Determinants

**Find $AB$ and $BA$, if possible.**

7. $A = \begin{bmatrix} 3 & 4 \\ -7 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 5 & 2 & -8 \\ -6 & 0 & 9 \end{bmatrix}$

*SOLUTION:*

$A = \begin{bmatrix} 3 & 4 \\ -7 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 5 & 2 & -8 \\ -6 & 0 & 9 \end{bmatrix}$

$A$ is a 2 × 2 matrix and $B$ is a 2 × 3 matrix. Because the number of columns of $A$ is equal to the number of rows of $B$, $AB$ exists.

To find the first entry of $AB$, find the sum of the products of the entries in row 1 of $A$ and column 1 of $B$. Follow the same procedure for row 2 column 1 of $AB$ and the remaining entries.

$$AB = \begin{bmatrix} 3(5)+4(-6) & 3(2)+4(0) & 3(-8)+4(9) \\ -7(5)+1(-6) & -7(2)+1(0) & -7(-8)+1(9) \end{bmatrix}$$
$$= \begin{bmatrix} 15-24 & 6+0 & -24+36 \\ -35-6 & -14+0 & 56+9 \end{bmatrix}$$
$$= \begin{bmatrix} -9 & 6 & 12 \\ -41 & -14 & 65 \end{bmatrix}$$

Because the number of columns of $B$ is *not* equal to the number of rows of $A$, $BA$ is undefined.

*ANSWER:*

$AB = \begin{bmatrix} -9 & 6 & 12 \\ -41 & -14 & 65 \end{bmatrix}$; $BA$ is undefined.

8. $A = \begin{bmatrix} 6 & -9 & 10 \\ 4 & 3 & 8 \end{bmatrix}$

$B = \begin{bmatrix} 6 & -8 \\ 3 & -9 \\ -2 & 5 \\ 4 & 1 \end{bmatrix}$

*SOLUTION:*

$A = \begin{bmatrix} 6 & -9 & 10 \\ 4 & 3 & 8 \end{bmatrix}$; $B = \begin{bmatrix} 6 & -8 \\ 3 & -9 \\ -2 & 5 \\ 4 & 1 \end{bmatrix}$

$A$ is a 2 × 3 matrix and $B$ is a 4 × 2 matrix. Because the number of columns of $A$ is *not* equal to the number of rows of $B$, $AB$ is undefined.

$B$ is a 4 × 2 matrix and $A$ is a 2 × 3 matrix. Because the number of columns of $B$ is equal to the number of rows of $A$, $BA$ exists.

To find the first entry of $BA$, find the sum of the products of the entries in row 1 of $B$ and column 1 of $A$. Follow the same procedure for row 2 column 1 of $BA$ and the remaining entries.

$$BA = \begin{bmatrix} 6(6)+(-8)(4) & 6(-9)+(-8)(3) & 6(10)+(-8)(8) \\ 3(6)+(-9)(4) & 3(-9)+(-9)(3) & 3(10)+(-9)(8) \\ -2(6)+5(4) & -2(-9)+5(3) & -2(10)+5(8) \\ 4(6)+1(4) & 4(-9)+1(3) & 4(10)+1(8) \end{bmatrix}$$

$$= \begin{bmatrix} 36-32 & -54-24 & 60-64 \\ 18-36 & -27-27 & 30-72 \\ -12+20 & 18+15 & -20+40 \\ 24+4 & -36+3 & 40+8 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -78 & -4 \\ -18 & -54 & -42 \\ 8 & 33 & 20 \\ 28 & -33 & 48 \end{bmatrix}$$

*ANSWER:*

$AB$ is undefined; $BA = \begin{bmatrix} 4 & -78 & -4 \\ -18 & -54 & -42 \\ 8 & 33 & 20 \\ 28 & -33 & 48 \end{bmatrix}$.

## 6-2 Matrix Multiplication, Inverses and Determinants

10. **CARS** The number of vehicles that a company manufactures each day from two different factories is shown, as well as the price of the vehicle during each sales quarter of the year. Use this information to determine which factory produced the highest sales in the 4th quarter.

| Factory | Model | | | |
|---|---|---|---|---|
| | Coupe | Sedan | SUV | Mini Van |
| 1 | 500 | 600 | 150 | 250 |
| 2 | 250 | 350 | 250 | 400 |

| Model | Quarter | | | |
|---|---|---|---|---|
| | 1st ($) | 2nd ($) | 3rd ($) | 4th ($) |
| Coupe | 18,700 | 17,100 | 16,200 | 15,600 |
| Sedan | 25,400 | 24,600 | 23,900 | 23,400 |
| SUV | 36,300 | 35,500 | 34,900 | 34,500 |
| Mini Van | 38,600 | 37,900 | 37,400 | 36,900 |

*SOLUTION:*

We need to determine the factory which produced the highest sales in the 4th quarter only. Therefore, the information we need from the 2nd table is the far right column.

For Factory 1, multiply the 1st row in the first table by the last column in the 2nd table. For Factory 2, multiply the 2nd row in the first table by the last column in the 2nd table.

Factory 1:

$$\begin{bmatrix} 500 & 600 & 150 & 250 \end{bmatrix} \cdot \begin{bmatrix} 15,600 \\ 23,400 \\ 34,500 \\ 36,900 \end{bmatrix}$$
$$= [500(15,600) + 600(23,400) + 150(34,500) + 250(36,900)]$$
$$= [36,240,000]$$

Factory 2:

$$\begin{bmatrix} 250 & 350 & 250 & 400 \end{bmatrix} \cdot \begin{bmatrix} 15,600 \\ 23,400 \\ 34,500 \\ 36,900 \end{bmatrix}$$
$$= [250(15,600) + 350(23,400) + 250(34,500) + 400(36,900)]$$
$$= [35,475,000]$$

Factory 1 which produced the highest sales in the 4th quarter.

*ANSWER:*

Factory 1

# 6-2 Matrix Multiplication, Inverses and Determinants

**Determine whether *A* and *B* are inverse matrices.**

19. $A = \begin{bmatrix} 12 & -7 \\ -5 & 3 \end{bmatrix}$

$B = \begin{bmatrix} 3 & 7 \\ 5 & 12 \end{bmatrix}$

*SOLUTION:*

$A = \begin{bmatrix} 12 & -7 \\ -5 & 3 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 7 \\ 5 & 12 \end{bmatrix}$

If $A$ and $B$ are inverse matrices, then $AB = BA = I$.

$$AB = \begin{bmatrix} 12(3) - 7(5) & 12(7) - 7(12) \\ -5(3) + 3(5) & -5(7) + 3(12) \end{bmatrix}$$
$$= \begin{bmatrix} 36 - 35 & 84 - 84 \\ -15 + 15 & -35 + 36 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$BA = \begin{bmatrix} 3(12) + 7(-5) & 3(-7) + 7(3) \\ 5(12) + 12(-5) & 5(-7) + 12(3) \end{bmatrix}$$
$$= \begin{bmatrix} 36 - 35 & -21 + 21 \\ 60 - 60 & -35 + 36 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Because $AB = BA = I$, $B = A^{-1}$ and $A = B^{-1}$.

*ANSWER:*

yes

# 6-2 Matrix Multiplication, Inverses and Determinants

20. $A = \begin{bmatrix} 4 & -5 \\ 5 & -6 \end{bmatrix}$

$B = \begin{bmatrix} -6 & 5 \\ -5 & 4 \end{bmatrix}$

*SOLUTION:*

$A = \begin{bmatrix} 4 & -5 \\ 5 & -6 \end{bmatrix}$; $B = \begin{bmatrix} -6 & 5 \\ -5 & 4 \end{bmatrix}$

If $A$ and $B$ are inverse matrices, then $AB = BA = I$.

$$AB = \begin{bmatrix} 4(-6)-5(-5) & 4(5)-5(4) \\ 5(-6)-6(-5) & 5(5)-6(4) \end{bmatrix}$$
$$= \begin{bmatrix} -24+25 & 20-20 \\ -30+30 & -25+24 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$BA = \begin{bmatrix} -6(4)+5(5) & -6(-5)+5(-6) \\ -5(4)+4(5) & -5(-5)+4(-6) \end{bmatrix}$$
$$= \begin{bmatrix} -24+25 & 30-30 \\ -20+20 & 25-24 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Because $AB = BA = I$, $B = A^{-1}$ and $A = B^{-1}$.

*ANSWER:*

yes

**Find the determinant of each matrix. Then find the inverse of the matrix, if it exists.**

35. $\begin{bmatrix} 6 & -5 \\ 3 & -2 \end{bmatrix}$

*SOLUTION:*

Find the determinant.

$\det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$$\begin{vmatrix} 6 & -5 \\ 3 & -2 \end{vmatrix} = 6(-2)-3(-5)$$
$$= -12+15$$
$$= 3$$

Because the determinant is not 0, the matrix is invertible. Find the inverse matrix.

$$A^{-1} = \frac{1}{\det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
$$= \frac{1}{3}\begin{bmatrix} -2 & 5 \\ -3 & 6 \end{bmatrix}$$
$$= \begin{bmatrix} -\frac{2}{3} & \frac{5}{3} \\ -1 & 2 \end{bmatrix}$$

Confirm that $AA^{-1} = A^{-1}A = I$.

$$AA^{-1} = \begin{bmatrix} 6 & -5 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} -\frac{2}{3} & \frac{5}{3} \\ -1 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 6\left(-\frac{2}{3}\right) - 5(-1) & 6\left(\frac{5}{3}\right) - 5(2) \\ 3\left(-\frac{2}{3}\right) - 2(-1) & 3\left(\frac{5}{3}\right) - 2(2) \end{bmatrix}$$
$$= \begin{bmatrix} -4+5 & 10-10 \\ -2+2 & 5-4 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} -\frac{2}{3} & \frac{5}{3} \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 6 & -5 \\ 3 & -2 \end{bmatrix}$$
$$= \begin{bmatrix} \left(-\frac{2}{3}\right)6 + \left(\frac{5}{3}\right)3 & \left(-\frac{2}{3}\right)(-5) + \left(\frac{5}{3}\right)(-2) \\ -1(6) + 2(3) & -1(-5) + 2(-2) \end{bmatrix}$$
$$= \begin{bmatrix} -4+5 & \frac{10}{3} - \frac{10}{3} \\ -6+6 & 5-4 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

*ANSWER:*

3; $\begin{bmatrix} -\frac{2}{3} & \frac{5}{3} \\ -1 & 2 \end{bmatrix}$

94. What are the dimensions of the matrix that results from the multiplication shown?

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \cdot \begin{bmatrix} j \\ k \\ l \end{bmatrix}$$

F 1 × 3
G 3 × 1
H 3 × 3
J 4 × 3

*SOLUTION:*

The first matrix is 3 rows by 3 columns, or 3 × 3. The second matrix is 3 rows by 1 column, or 3 × 1. The multiplication of these will produce a matrix with the rows of the first matrix (3) and the columns of the second matrix (1). Therefore, the correct choice is G.

*ANSWER:*

G