5.4 Review WS

1. $\cos\frac{7\pi}{12} = \cos\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right) = \cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

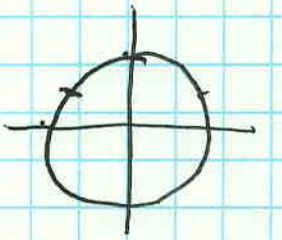
$= \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}}$

2. $\sin 165 = \sin(210-45)$

$\sin 210\cos 45 - \cos 210\sin 45$

$\left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{-\sqrt{2}}{4} + \frac{\sqrt{6}}{4}}$



3. $\tan 195 = \tan(150°+45°)$

$\frac{\tan 150 + \tan 45}{1 - \tan 150\tan 45} \quad \frac{\left(\frac{\sqrt{3}}{3}\right)+1}{1-\left(\frac{\sqrt{3}}{3}\right)(1)}$

$\frac{\left(\frac{\sqrt{3}}{3}+1\right)}{\left(1+\frac{\sqrt{3}}{3}\right)} = \frac{\left(\frac{3+\sqrt{3}}{3}\right)}{\left(\frac{3+\sqrt{3}}{3}\right)}\left(\frac{3}{3+\sqrt{3}}\right) = \frac{3+\sqrt{3}}{3+\sqrt{3}}\frac{(3+\sqrt{3})}{(3+\sqrt{3})}$

$= \frac{9+6\sqrt{3}+3}{9-3} = \frac{12+6\sqrt{3}}{6} = \boxed{2+\sqrt{3}}$

4. $\sin\frac{17\pi}{12} = \sin\left(\frac{2\pi}{12}+\frac{15\pi}{12}\right) \cdot \sin\left(\frac{\pi}{6}+\frac{5\pi}{4}\right)$

$\sin\frac{\pi}{6}\cos\frac{5\pi}{4} + \cos\frac{\pi}{6}\sin\frac{5\pi}{4}$

$\left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = \boxed{\frac{-\sqrt{2}}{4} - \frac{\sqrt{6}}{4}}$

5. $\tan 15^\circ = \tan(60-45)$

$$\frac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45} = \frac{(\sqrt{3}) - 1}{1 + \sqrt{3} \cdot 1} = \frac{\sqrt{3}-1}{\sqrt{3}+1}\,\frac{(\sqrt{3}+1)}{(\sqrt{3}-1)}$$

$$= \frac{3 - 2\sqrt{3} + 1}{3 - 1} = \frac{4 - 2\sqrt{3}}{2} = \boxed{2 - \sqrt{3}}$$

6. $\sin 20 \cos 10 + \cos 20 \sin 10$

$= \sin(20+10) = \sin 30^\circ = \boxed{\frac{1}{2}}$

7. $\dfrac{\tan\frac{\pi}{9} + \tan\frac{5\pi}{36}}{1 - \tan\frac{\pi}{9}\tan\frac{5\pi}{36}} = \tan\left(\frac{\pi}{9} + \frac{5\pi}{36}\right) = \tan\left(\frac{4\pi}{36} + \frac{5\pi}{36}\right)$

$= \tan\left(\frac{9\pi}{36}\right) = \boxed{\tan\left(\frac{\pi}{4}\right) = 1}$

8. $\cos\frac{9\pi}{8}\cos\frac{5\pi}{24} - \sin\frac{9\pi}{8}\sin\frac{5\pi}{24} = \cos\left(\frac{9\pi}{8} + \frac{5\pi}{24}\right)$

$= \cos\left(\frac{27\pi}{24} + \frac{5\pi}{24}\right) = \cos\left(\frac{32\pi}{24}\right) = \boxed{\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}}$

9. $\cos 7x \cos 2x - \sin 7x \sin 2x = \cos(7x+2x) = \boxed{\cos 9x}$

10. $\sin 188^\circ \cos 53^\circ - \cos 188^\circ \sin 53^\circ = \sin(188-53) = \sin 135 = \boxed{\frac{\sqrt{2}}{2}}$

11. $\cos 8x \sin 6x - \sin 8x \cos 6x =$

assoc. prop. of mult. → $\sin 6x \cos 8x - \cos 6x \sin 8x$ ~OR~ $-1(\sin 8x \cos 6x - \cos 8x \sin 6x)$

$= \sin(6x-8x)$
$= \sin(-2x)$
$= -\sin 2x$ (odd/even identity)

$= -1(\sin(8x-6x))$
$= -\sin 2x$

factor -1 out to switch order

12. $\sin(360° + \theta) = \sin\theta$

$\sin 360° \cos\theta + \cos 360° \sin\theta =$

$0 \cos\theta + 1 \sin\theta = \sin\theta$

13. $\sec(180° - \theta) = -\sec\theta$

$$\frac{1}{\cos(180° - \theta)}$$

$$= \frac{1}{\cos 180° \cos\theta + \sin 180 \sin\theta}$$

$$= \frac{1}{-1\cos\theta + 0 \sin\theta}$$

$$= \frac{1}{-\cos\theta} = -\sec\theta$$

14. $\cos\left(\frac{5\pi}{4} + x\right) + \sin\left(\frac{5\pi}{4} - x\right) = 0$

$\cos\frac{5\pi}{4}\cos x - \sin\frac{5\pi}{4}\sin x + \sin\frac{5\pi}{4}\cos x - \cos\frac{5\pi}{4}\sin x = 0$

$\left(\frac{-\sqrt{2}}{2}\right)\cos x - \left(\frac{-\sqrt{2}}{2}\right)\sin x + \left(\frac{-\sqrt{2}}{2}\right)\cos x - \left(\frac{-\sqrt{2}}{2}\right)\sin x = 0$

$-\sqrt{2}\cos x + \sqrt{2}\sin x = 0$

$\sin x = \cos x$ $\quad \boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$

15. $\sin\left(\frac{2\pi}{3}-x\right)+\sin\left(\frac{2\pi}{3}+x\right)=0$

$\sin\frac{2\pi}{3}\cos x-\cos\frac{2\pi}{3}\sin x+\sin\frac{2\pi}{3}\cos x+\cos\frac{2\pi}{3}\sin x$

$\left(\frac{\sqrt{3}}{2}\right)\cos x-\left(-\frac{1}{2}\right)\sin x+\left(\frac{\sqrt{3}}{2}\right)\cos x+\left(-\frac{1}{2}\right)\sin x=0$

$\frac{\sqrt{3}}{2}\cos x+\frac{\sqrt{3}}{2}\cos x=0$

$\sqrt{3}\cos x=0$

$\cos x=0$ ~~$\cos x$~~ $x=$,

16. $\sin\left(\frac{3\pi}{2}+x\right)-\cos\left(\frac{\pi}{2}+x\right)=0$ distribute

$\sin\frac{3\pi}{2}\cos x+\cos\frac{3\pi}{2}\sin x-\left(\cos\frac{\pi}{2}\cos x-\sin\left(\frac{\pi}{2}\right)\sin x\right)=0$

$(-1)\cos x+0\sin x-0\cos x+(1)\sin x=0$

$-\cos x+\sin x=0$ $\sin x=\cos x$ $x=\frac{\pi}{4},\frac{5\pi}{4}$