5-4   Study Guide and Intervention   *(continued)*

Sum and Difference Identities

Solve Trigonometric Equations You can solve trigonometric equations using the sum and difference identities along with algebraic methods and the same techniques you used before.

Example Find the solutions of $\sin\left(\frac{\pi}{2} + x\right) + \cos\left(\frac{\pi}{2} + x\right) = 0$ on the interval $[0, 2\pi)$.

$$\sin\left(\frac{\pi}{2} + x\right) + \cos\left(\frac{\pi}{2} + x\right) = 0 \qquad \text{Original equation}$$

$$\sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x + \cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x = 0 \qquad \text{Cosine Sum Identity}$$

$$1(\cos x) + 0(\sin x) + 0(\cos x) - 1(\sin x) = 0 \qquad \text{Substitute.}$$

$$\cos x - \sin x = 0 \qquad \text{Simplify.}$$

$$\cos x = \sin x \qquad \text{Add.}$$

On the interval $[0, 2\pi)$, $\cos x = \sin x$ when $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$.

Exercises

Find the solution of each equation on the interval $[0, 2\pi)$.

1. $\cos\left(\frac{\pi}{4} - x\right) - \sin\left(\frac{\pi}{4} - x\right) = -1$

2. $\sin(\pi + x) + \sin(\pi + x) = 1$

3. $\cos\left(\frac{3\pi}{2} + x\right) + \sin\left(\frac{3\pi}{2} - x\right) = 0$

4. $\tan(\pi - x) + \tan(\pi - x) = -2$

5. $\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$

6. $\cos\left(x + \frac{\pi}{6}\right) - 1 = \cos\left(x - \frac{\pi}{6}\right)$

5-4 Practice

Sum and Difference Identities

Find the exact value of each trigonometric expression.

1. $\cos \frac{5\pi}{12}$

2. $\sin(-165°)$

3. $\tan 345°$

4. $\csc 915°$

5. $\tan\left(-\frac{7\pi}{12}\right)$

6. $\sec \frac{\pi}{12}$

Simplify each expression.

7. $\cos \frac{3\pi}{2} \cos \pi - \sin \frac{3\pi}{2} \sin \pi$

8. $\dfrac{\tan 30° - \tan x}{1 + \tan 30° \tan x}$

Write each trigonometric expression as an algebraic expression.

9. $\sin(\arccos x + \arcsin x)$

10. $\cos\left(\arccos \frac{1}{2} - \arcsin x\right)$

Verify each cofunction or reduction identity.

11. $\sin(360° + \theta) = \sin \theta$

12. $\cos(180° - \theta) = -\cos \theta$

Find the solutions to each expression on the interval $[0, 2\pi)$.

13. $\cos\left(\frac{5\pi}{4} + x\right) + \sin\left(\frac{5\pi}{4} - x\right) = 0$

14. $\sin\left(\frac{2\pi}{3} - x\right) + \sin\left(\frac{2\pi}{3} + x\right) = 0$

15. Sound waves can be modeled by the equations of the form $y_1 = 20 \sin(3x + \theta)$. A wave traveling in the opposite direction can be modeled by $y_2 = 20 \sin(3x - \theta)$. Show that $y_1 + y_2 = 40 \sin 3x \cos \theta$.

Lesson 5-4