5-4 Study Guide

1. $\cos\left(\frac{\pi}{4} - x\right) - \sin\left(\frac{\pi}{4} + x\right) = -1$

$\cos\frac{\pi}{4}\cos x + \sin\frac{\pi}{4}\sin x - \left(\sin\frac{\pi}{4}\cos x + \cos\frac{\pi}{4}\sin x\right)$

$\frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x$

$\frac{\sqrt{2}}{2}\sin x + \frac{\sqrt{2}}{2}\sin x = -1$

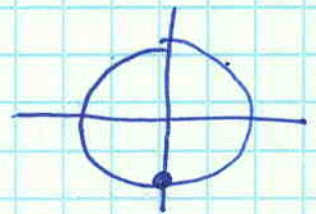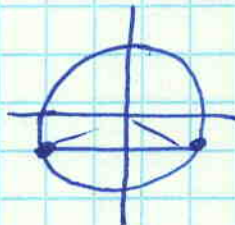$\sqrt{2}\sin x = -1 \quad \sin x = -\frac{\sqrt{2}}{2} \quad \boxed{x = \frac{5\pi}{4}, \frac{7\pi}{4}}$

2. $\sin\left(\frac{\pi}{2} + x\right) + \sin\left(\pi + x\right) = 1$

$\sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x + \sin\pi\cos x + \cos\pi\sin x = 1$

$(0)\cos x + (-1)\sin x + (0)\cos x + (-1)\sin x = 1$

$-\sin x - \sin x = 1 \quad -2\sin x = 1 \quad \sin x = -\frac{1}{2}$

$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

3. $\cos\left(\frac{3\pi}{2} + x\right) + \sin\left(\frac{3\pi}{2} - x\right) = 0$

$\cos\frac{3\pi}{2}\cos x - \sin\frac{3\pi}{2}\sin x + \sin\frac{3\pi}{2}\cos x - \cos\frac{3\pi}{2}\sin x = 0$

$0\cos x - (-1)\sin x + (-1)\cos x - 0\sin x = 0$

$\sin x - \cos x = 0 \quad \sin x = \cos x \quad \boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$

4. $\tan(\pi - x) + \tan(\pi - x) = -2$

$$\frac{\tan\pi - \tan x}{1 + \tan\pi\tan x} + \frac{\tan\pi - \tan x}{1 + \tan\pi\tan x} = -2$$

$$\frac{2\tan\pi - 2\tan x}{1 + \tan\pi\tan x} = -2 \qquad \frac{2(0) - 2\tan x}{1 + 0\tan x} = -2$$

$$-2\tan x = -2 \quad \tan x = 1 \quad \boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$$

5. $\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$

$$\sin x \cos\frac{\pi}{3} + \cos x \sin\frac{\pi}{3} + \sin x \cos\frac{\pi}{3} - \cos x \sin\frac{\pi}{3} = 1$$

$$2\sin x \cos\frac{\pi}{3} = 1$$

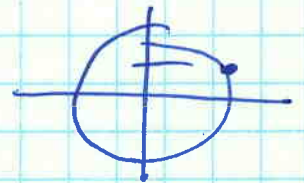$$2\left(\frac{1}{2}\right)\sin x = 1 \qquad \sin x = 1 \quad \boxed{x = \frac{\pi}{2}}$$

6. $\cos\left(x + \frac{\pi}{6}\right) - \cos\left(x - \frac{\pi}{6}\right) = 1$

$$\cos x \cos\frac{\pi}{6} - \sin x \sin\frac{\pi}{6} - \left(\cos x \cos\frac{\pi}{6} + \sin x \sin\frac{\pi}{6}\right) = 1$$

$$-2\sin x \sin\frac{\pi}{6} = 1$$

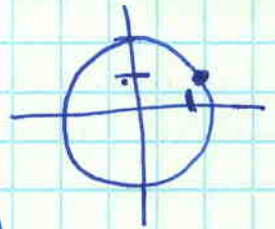$$-2\left(\frac{1}{2}\right)\sin x = 1 \qquad \sin x = -1 \quad \boxed{x = \frac{3\pi}{2}}$$

5-4 Practice WS

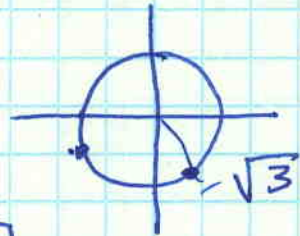1. $\cos\frac{5\pi}{12} = \cos\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right) = \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

$= \cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4} = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

2. $\sin(-165°) = \sin(210° - 45°)$

$= \sin 210°\cos 45° - \cos 210°\sin 45°$

$\left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{-\sqrt{2} + \sqrt{6}}{4}}$
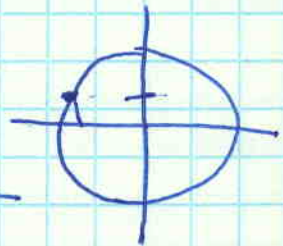
3. $\tan(345°) = \tan(300° + 45°)$

$= \dfrac{\tan 300 + \tan 45}{1 - \tan 300 \tan 45} = \dfrac{(-\sqrt{3}) + 1}{1 - (-\sqrt{3})(1)} = \dfrac{1 - \sqrt{3}}{1 + \sqrt{3}}$

$\dfrac{1 - \sqrt{3}}{1 + \sqrt{3}} \cdot \dfrac{(1 - \sqrt{3})}{(1 + \sqrt{3})} = \dfrac{1 - 2\sqrt{3} + 3}{1 - 3} = \dfrac{4 - 2\sqrt{3}}{-2} = \boxed{-2 + \sqrt{3}}$

4. $\csc(915) = \csc 195 = \csc(150° + 45)$

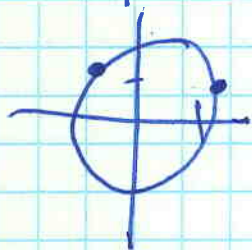$= \dfrac{1}{\sin(150 + 45)} = \dfrac{1}{\sin 150 \cos 45 + \cos 150 \sin 45}$

$= \dfrac{1}{\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)} = \dfrac{1}{\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}} = \dfrac{4}{\sqrt{2} - \sqrt{6}}$

$\dfrac{4}{\sqrt{2} - \sqrt{6}} \cdot \dfrac{(\sqrt{2} + \sqrt{6})}{(\sqrt{2} + \sqrt{6})} = \dfrac{4\sqrt{2} + 4\sqrt{6}}{2 - 6} = \boxed{-\sqrt{2} - \sqrt{6}}$

5. $\tan\left(-\dfrac{7\pi}{12}\right) = \tan\left(\dfrac{2\pi}{12} - \dfrac{9\pi}{12}\right) = \tan\left(\dfrac{\pi}{6} - \dfrac{3\pi}{4}\right)$

$$\dfrac{\tan\frac{\pi}{6} - \tan\frac{3\pi}{4}}{1 + \tan\frac{\pi}{6}\tan\frac{3\pi}{4}} = \dfrac{\left(\frac{\sqrt{3}}{3}\right) - (-1)}{1 + \left(\frac{\sqrt{3}}{3}\right)(-1)} = \dfrac{\left(\frac{\sqrt{3}+3}{3}\right)}{\left(\frac{3-\sqrt{3}}{3}\right)}$$
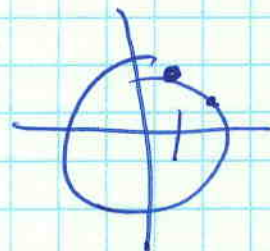
$$\dfrac{3+\sqrt{3}}{3-\sqrt{3}} \cdot \dfrac{(3+\sqrt{3})}{(3+\sqrt{3})} = \dfrac{9+6\sqrt{3}+3}{9-3} = \dfrac{12+6\sqrt{3}}{6}$$

$$= \boxed{2+\sqrt{3}}$$

6. $\sec\dfrac{\pi}{12} = \sec\left(\dfrac{4\pi}{12} - \dfrac{3\pi}{12}\right) = \sec\left(\dfrac{\pi}{3} - \dfrac{\pi}{4}\right)$

$$= \dfrac{1}{\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)} = \dfrac{1}{\cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)}$$

$$= \dfrac{1}{\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)} = \dfrac{1}{\frac{\sqrt{2}+\sqrt{6}}{4}} = \dfrac{4}{\sqrt{2}+\sqrt{6}}$$

$$\dfrac{4\sqrt{2}-4\sqrt{6}}{8} = \boxed{\dfrac{\sqrt{2}-\sqrt{6}}{2}}$$

7. $\cos\frac{3\pi}{2}\cos\pi - \sin\frac{3\pi}{2}\sin\pi$

$= \cos\left(\frac{3\pi}{2}+\pi\right) = \cos\frac{5\pi}{2} = \boxed{\cos\frac{\pi}{2} = 0}$

8. $\dfrac{\tan 30° - \tan x}{1+\tan 30° \tan x} = \boxed{\tan(30°-x)}$

9. $\sin(\arccos x + \arcsin x)$

$= \sin(\arccos x)\cos(\arcsin x) + \cos(\arccos x)\sin(\arcsin x)$

$\boxed{\sin(\arccos x)(\cos(\arcsin x)) = x^2}$ $\quad x \quad x$

10 $\cos\left(\arccos\frac{1}{2} - \arcsin x\right)$

$= \cos\left(\arccos\frac{1}{2}\right)\cos(\arcsin x) + \sin\left(\arccos\frac{1}{2}\right)(\sin(\arcsin x))$

$\cos\frac{\pi}{3}\cos(\arcsin x) + \sin\left(\frac{\pi}{3}\right)\sin(\arcsin x)$

$\boxed{\dfrac{1}{2}\cos(\arcsin x) + \dfrac{\sqrt{3}}{2} x}$

11. $\sin(360° + \theta) = \sin\theta$

$\sin 360\cos\theta + \cos 360\sin\theta =$

$\quad 0 \cos\theta + 1 \sin\theta = \sin\theta$
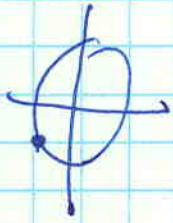
12. $\cos(180° - \theta) = -\cos\theta$

$\cos(180°)(\cos\theta) + \sin 180\sin\theta$

$(-1)\cos\theta + 0\sin\theta$

$\boxed{= -\cos\theta}$

13. $\cos\left(\frac{5\pi}{4}+x\right) + \sin\left(\frac{5\pi}{4}-x\right) = 0$

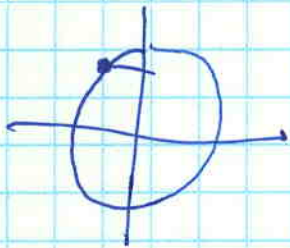$\cos\frac{5\pi}{4}\cos x - \sin\frac{5\pi}{4}\sin x + \sin\frac{5\pi}{4}\cos x - \cos\frac{5\pi}{4}\sin x = 0$

$\left(\frac{-\sqrt{2}}{2}\right)\cos x - \left(-\frac{\sqrt{2}}{2}\right)\sin x + \left(-\frac{\sqrt{2}}{2}\right)\cos x - \left(-\frac{\sqrt{2}}{2}\right)\sin x$

$\frac{2\sqrt{2}}{2}\cos x + \frac{2\sqrt{2}}{2}\sin x = 0$

$\cos x = \sin x$ $\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$

14. $\sin\left(\frac{2\pi}{3}-x\right) + \sin\left(\frac{2\pi}{3}+x\right) = 0$

$\sin\frac{2\pi}{3}\cos x - \cos\frac{2\pi}{3}\sin x + \sin\frac{2\pi}{3}\cos x + \cos\frac{2\pi}{3}\sin x = 0$

$2\sin\frac{2\pi}{3}\cos x = 0$

$2\left(\frac{\sqrt{3}}{2}\right)\cos x = 0$ $\qquad \cos x = 0$

$\boxed{x = 0, \pi}$

15. $y_1 = 20\sin(3x+\theta)$ $\quad y_2 = 20\sin(3x-\theta)$ $\quad y_1+y_2 = 40\sin 3x\cos\theta$

$20\sin(3x+\theta) + 20\sin(3x-\theta)$

$20\left[(\sin 3x\cos\theta + \cos 3x\sin\theta)\right] + 20\left[(\sin 3x\cos\theta - \cos 3x\sin\theta)\right]$

$20\sin 3x\cos\theta + 20\cos 3x\sin x + 20\sin 3x\cos\theta - 20\cos 3x\sin\theta$

$\boxed{40\sin 3x\cos\theta}$