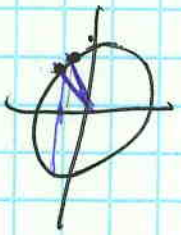
341 4, 5, 15, 20, 21, 33, 36-38.

4. $\cos\frac{17\pi}{12}$ $\quad \frac{9}{12}+\frac{8}{12}$ $\quad \cos\left(\frac{3\pi}{4}+\frac{2\pi}{3}\right)$



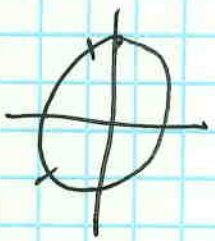
$= \cos\left(\frac{3\pi}{4}\right)\left(\cos\frac{2\pi}{3}\right) - \sin\left(\frac{3\pi}{4}\right)\left(\sin\left(\frac{2\pi}{3}\right)\right.$

$\left(\frac{-\sqrt{2}}{2}\right)\left(\frac{-1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

5. $\tan\frac{23\pi}{12}$ $\quad \frac{12}{12}$ $\quad \frac{11}{12}$ $= \frac{12}{12}+\frac{3}{12}+\frac{8}{12}$

$= \frac{15}{12}+\frac{8}{12}$ $= \tan\left(\frac{5\pi}{4}+\frac{2\pi}{3}\right)$



$= \dfrac{\tan\left(\frac{5\pi}{4}\right)+\tan\left(\frac{2\pi}{3}\right)}{1-\tan\left(\frac{5\pi}{4}\right)\left(\tan\left(\frac{2\pi}{3}\right)\right.}$

$\frac{+1+(-\sqrt{3})}{1-(1)(-\sqrt{3})} = \frac{1-\sqrt{3}}{1+\sqrt{3}}\cdot\frac{(1-\sqrt{3})}{(1-\sqrt{3})}$

$= \frac{1-2\sqrt{3}+3}{1-3} = \frac{4-2\sqrt{3}}{-2} = \boxed{\sqrt{3}-2}$

15 $\sin\frac{\pi}{3}\cos\frac{\pi}{12} - \cos\frac{\pi}{3}\sin\frac{\pi}{12}$

$\sin\left(\frac{\pi}{3}-\frac{\pi}{12}\right) = \sin\left(\frac{4\pi}{12}-\frac{\pi}{12}\right) = \sin\left(\frac{3\pi}{12}\right) = \sin\left(\frac{\pi}{4}\right)$

$= \boxed{\frac{\sqrt{2}}{2}}$

20. $\cos 2x \sin x - \sin 2x \cos x$

$= -1(\sin 2x \cos x - \cos 2x \sin x)$

$= -\sin(2x-x) = \boxed{-\sin x}$

$\sin x \cos 2x - \cos x \sin 2x$

$\sin(x-2x)$

$\sin(-x)$

$= -\sin x$

21. $\cos x \cos 2x + \sin x \sin 2x = \cos(x-2x)$

$= \boxed{\cos(-x)}$

33. $\sec\left(\frac{\pi}{2}-x\right) = \csc x$

$= \dfrac{1}{\cos\left(\frac{\pi}{2}-x\right)}$

$= \dfrac{1}{\cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x}$

$= \dfrac{1}{0(\cos x) + 1(\sin x)}$

$= \dfrac{1}{\sin x} = \csc x$ ✓

36. $\cos(2\pi+\theta) = \cos\theta$ $\quad \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \cos 2\pi\cos\theta - \sin 2\pi\sin\theta$

$= 1\cos\theta - 0\sin\theta$

$= \cos\theta$ ✓

37. $\sin(\pi-\theta) = \sin\theta$ $\quad \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \sin\pi\cos\theta - \cos\pi\sin\theta$

$0\cos\theta - (-1)\sin\theta$

$= \sin\theta$ ✓

38. $\sin(90^\circ+\theta) = \cos\theta$

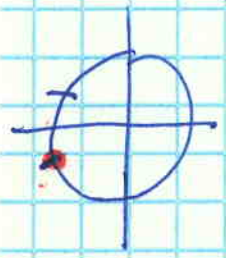
$= \sin 90^\circ\cos\theta + \cos 90^\circ\sin\theta$

$= 1\cos\theta + 0\sin\theta$

$= \cos\theta$ ✓

#5 $\tan\frac{23\pi}{12} = \tan\left(\frac{14\pi}{12}+\frac{9\pi}{12}\right) = \tan\left(\frac{7\pi}{6}+\frac{3\pi}{4}\right)$

$$\frac{\tan\frac{7\pi}{6}+\tan\frac{3\pi}{4}}{1-\tan\left(\frac{2\pi}{6}\right)\left(\tan\left(\frac{3\pi}{4}\right)\right)} = \frac{\left(+\frac{\sqrt{3}}{3}\right)+(-1)}{1-\left(+\frac{\sqrt{3}}{3}\right)(-1)}$$



$$= \frac{\left(\frac{3+\sqrt{3}}{3}\right)}{\left(\frac{3+\sqrt{3}}{3}\right)}$$

$$\frac{\frac{\sqrt{3}}{3}+(-1)}{1-\left(\frac{\sqrt{3}}{3}\right)(-1)}$$

$$\frac{\frac{\sqrt{3}}{3}-1}{1+\frac{\sqrt{3}}{3}} = \frac{\left(\frac{\sqrt{3}-3}{3}\right)}{\left(\frac{3+\sqrt{3}}{3}\right)} \sim = \frac{\sqrt{3}-3}{\sqrt{3}+3}\cdot\frac{(\sqrt{3}-3)}{(\sqrt{3}-3)}$$

$$\frac{3-6\sqrt{3}+9}{3-9}$$

$$\frac{12-6\sqrt{3}}{-6}$$

$$-2+\sqrt{3}$$

$$\tan\left(\frac{\pi}{4}+\frac{5\pi}{3}\right)$$

$$1-\sqrt{3}$$