331: 21-26, 45

21. $1 = \cot^2 x + \csc x$
$1 = \csc^2 x - 1 + \csc x$
$2 = \csc^2 x + \csc x$

$\cot^2 + 1 = \csc^2$
$\quad -1 \quad -1$

$0 = \csc^2 x + \csc x - 2$
$(\csc x + 2)(\csc x - 1) = 0$

$x^2 + x - 2$
$(x+2)(x-1)$

$\csc x = -2 \qquad \csc x = 1 \rightarrow \sin = 1$

$\sin = -\frac{1}{2}$ $\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6} \qquad x = \frac{\pi}{2}}$ ✓ checked graphically

22. $\sec x = \tan x + 1 \qquad [0, 2\pi)$

$(\sec x)^2 = (\tan x + 1)^2$
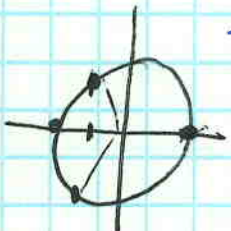
Check

$\sec^2 x = \tan^2 x + 2\tan x + 1$

$\tan^2 x + 1 = \tan^2 x + 2\tan x + 1$
$-\tan^2 x - 1 \quad -\tan^2 x \qquad -1$

$\sec 0 = \tan 0 + 1$
$1 \overset{?}{=} 0 + 1$

$\sec \pi = \tan \pi + 1$
$-1 = 0 + 1$ X

$0 = 2\tan x \qquad \boxed{x = 0, \cancel{\pi}}$
$0 = \tan x$

23. $\tan^2 x = 1 - \sec x \qquad [0, 2\pi)$
$\sec^2 x - 1 = 1 - \sec x$
$\quad +1 \quad +1 + \sec x$
$\sec^2 x + \sec x - 2 = 0$

$(\sqrt{3})^2 = 1 - (-2)$ ✓
$(\sqrt{3})^2 = 1 - (-2)$ ✓

$(\sec x + 2)(\sec x - 1) = 0$

$1 = 0$

$\cos = -\frac{1}{2}$ $\sec x = -2 \qquad \sec x = 1$ $\frac{1}{\cos 0} = \frac{1}{1}$ $\tan^2 0 = 1 - \sec 0$
$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$ ✓ ✓ $x = 0$ undef $1 = 1 - 1$
$1 = 0$

24. $| \csc x + \cot x = (1)^2$

$\csc^2 x + 2\csc x \cot x + \cot^2 x = 1$

* Different tactic...

$(\csc x)^2 = (1 - \cot x)^2$

$\csc^2 x = 1 - 2\cot x + \cot^2 x$

$\underline{\cot^2 x + 1} = \underline{1 + \cot^2 x - 2\cot x}$

$0 = -2\cot x$

$0 = \cot x$

$\tan = \frac{1}{0}$

$\boxed{x = \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}}$

csc   cot

$\frac{1}{\sin} \cdot \frac{\cos}{\sin}$

check

$\csc \frac{\pi}{2} + \cot(\frac{\pi}{2})$

$1 + 0 = 1 \checkmark$

$\csc \frac{3\pi}{2} + \cot \frac{3\pi}{2}$

$-1 + 0 \neq +1$

$\times$

25. $2 - 2\cos^2 x = \sin x + 1$

$2 - 2(1 + \sin^2 x) = \sin x + 1$

$\underset{0}{\underline{2 - 2}} + 2\sin^2 x = \sin x + 1$

$\cancel{+} \; 2\sin^2 x = \sin x + 1$

$0 = 2\sin^2 x + \sin x \cancel{+1}$

$(2\sin x + 1)(\sin x - 1) = 0$

$2\sin x = 1 \qquad \sin x = 1$

$\qquad\qquad\qquad x = \frac{\pi}{2}$

$\sin x = -\frac{1}{2}$

$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}}$

Check

$2 - 2\left(-\frac{\sqrt{3}}{2}\right)^2 = -\frac{1}{2} + 1$

$2 - 2\left(\frac{3}{4}\right) = \frac{1}{2}$

$2 - \frac{6}{4} = \frac{1}{2} \checkmark$

$2 - 2\left(+\frac{\sqrt{3}}{2}\right)^2 = -\frac{1}{2} + 1$

$2 - \frac{6}{4} = \frac{1}{2} \checkmark$

$2 - 2(0) = 1 + 1 \checkmark$

26 $\cos x - 4 = \sin x - 4$

$\cos x = \sin x$
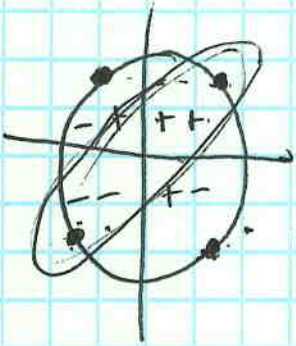
$\cos^2 x = \sin^2 x$

$\cos^2 x = 1 - \cos^2 x$

$2\cos^2 x = 1$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} = \frac{\pm\sqrt{2}}{2}$

$\boxed{x = \frac{\pi}{4}, \; \frac{3\pi}{4} \; X}$ $\boxed{\frac{3\pi}{4} \; \frac{5\pi}{4} \checkmark}$

Check

$\cos\frac{\pi}{4} - 4 = \sin\frac{\pi}{4} - 4$

$\frac{\sqrt{2}}{2} - 4 = \frac{\sqrt{2}}{2} - 4 \checkmark$

$\cos\frac{7\pi}{4} = \sin\frac{7\pi}{4}$

$\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2} \; X$

$\cos\frac{3\pi}{4} = \sin\frac{3\pi}{4}$

$\frac{-\sqrt{2}}{2} \neq \frac{\sqrt{2}}{2}$

$\cos\frac{5\pi}{4}$

$\frac{\sqrt{2}}{2} \neq -\frac{\sqrt{2}}{2}$

44. $y = \tan^3 x - \tan x$ x-intercepts

$0 = \tan^3 x - \tan x$

$0 = \tan x (\tan^2 x - 1)$

$\tan x = 0 \qquad \tan^2 x = 1$

$\qquad\qquad\qquad \tan x = \pm 1$

$x = 0, \pi \qquad x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

45. $4\tan x = 2\sec^2 x$ $\qquad [0, 4\pi)$

$\qquad\qquad t + 1 = s$

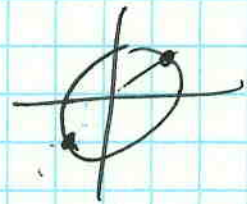$2\tan x = \sec^2 x$

$2\tan x = \tan^2 x + 1$

$\qquad 0 = \tan^2 x - 2\tan x + 1$

$\qquad 0 = (\tan x - 1)^2$

$\qquad 0 = (\tan x - 1)(\tan x - 1)$

$\qquad 0 = \quad \tan x = +1$

$\qquad\qquad x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{9\pi}{4}, \dfrac{13\pi}{4}$

$\qquad x^2 - 2x + 1$
$\qquad (x - 1)(x - 1)$

$\qquad 4\tan\left(\dfrac{5\pi}{4}\right) = 2\left(\sec\dfrac{5\pi}{4}\right)^2$

$\qquad\qquad 4 = 2 \cdot 2$

$\qquad\qquad 4 = 2 \cdot 2 =$

$\qquad \dfrac{\sqrt{2}}{2} \quad \left(\dfrac{2}{\sqrt{2}}\right)\left(\dfrac{4}{2}\right)$

$\qquad \left(-\dfrac{2}{\sqrt{2}}\right)$