

331: 3, 5, 8-10, 13-17

3. $2 = 4 \cos^2 X + 1$

$4 \cos^2 X = 1$

$\cos^2 X = \frac{1}{4}$

$\cos X = \pm \frac{1}{2}$

on $[0, 2\pi)$ $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

on $(-\infty, \infty)$ $x = \frac{\pi}{3} + 2\pi n$ $x = \frac{4\pi}{3} + 2\pi n$

$x = \frac{2\pi}{3} + 2\pi n$ $x = \frac{5\pi}{3} + 2\pi n$

5. $9 + \cot^2 X = 12$

$\cot^2 X = 3$

$\cot X = \pm \sqrt{3}$

($\tan X = \pm \frac{1}{\sqrt{3}}$)

on $[0, \pi)$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$

on $(-\infty, \infty)$ $x = \frac{\pi}{6} + \pi n$ and $x = \frac{5\pi}{6} + \pi n$

8. $11 = 3(\csc^2 X + 7)$

$4 = 3 \csc^2 X$

$\frac{4}{3} = \csc^2 X$

$\pm \frac{2}{\sqrt{3}} = \csc X$

$\hookrightarrow (\sin = \pm \frac{\sqrt{3}}{2})$

on $[0, 2\pi)$ $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

on $(-\infty, \infty)$ $x = \frac{\pi}{3} + 2\pi n, x = \frac{2\pi}{3} + 2\pi n$

$x = \frac{4\pi}{3} + 2\pi n, x = \frac{5\pi}{3} + 2\pi n$

9. $6 \tan^2 X - 2 = 4$

$6 \tan^2 X = 6$

$\tan^2 X = 1$

$\tan X = \pm 1$

~~10~~

on $[0, \pi)$ $x = \frac{\pi}{4}, \frac{3\pi}{4}$

on $(-\infty, \infty)$ $x = \frac{\pi}{4} + \pi n, x = \frac{3\pi}{4} + \pi n$

10. $9 + \sin^2 X = 10$

$\sin^2 X = 1$

$\sin X = \pm 1$

on $[0, 2\pi)$, $x = \frac{\pi}{2}, \frac{3\pi}{2}$

on $(-\infty, \infty)$ $x = \frac{\pi}{2} + 2\pi n, x = \frac{3\pi}{2} + 2\pi n$

$$13. \sin^4 x + 2\sin^2 x - 3 = 0$$

$$x^2 + 2x - 3 = 0$$

$$(x-1)(x+3) = 0$$

$$(\sin^2 x - 1)(\sin^2 x + 3) = 0$$

$$\sin^2 x = +1$$

$$\sin x = \pm \sqrt{1}$$

$$\sin^2 x = -3$$

$$\sin x = \pm \sqrt{-3}$$

no solution

on $[0, 2\pi)$



$$\boxed{\begin{array}{l} \sin x = \pm 1 \\ x = \frac{\pi}{2}, \frac{3\pi}{2} \end{array}}$$

$$14. -2\sin x = -\sin x \cos x$$

on $[0, 2\pi)$

$$-2\sin x + \sin x \cos x = 0$$

$$\sin x (-2 + \cos x) = 0$$

$$\sin x = 0$$

$$x = 0, \pi$$

$$\cos x = 2$$

no solution

$$15. 4 \cot x = \cot x \sin^2 x$$

$$0 = \cot x \sin^2 x - 4 \cot x$$

$$0 = \cot x (\sin^2 x - 4)$$

$$0 = \cot x (\sin x + 2)(\sin x - 2)$$

$$\cot x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sin x = -2$$

no solution

$$\sin x = 2$$

no solution

$$16. \csc^2 x - \csc x + 9 = 11$$

$$\csc^2 x - \csc x - 2 = 0$$

$$(\csc x + 1)(\csc x - 2) = 0$$

$$\csc x = -1$$

$$\sin x = -1$$

$$\csc x = 2$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{3\pi}{2}$$

$$x = \frac{\pi}{6}$$

$$x = \frac{5\pi}{6}$$

$$17: \cos^3 x + \cos^2 x - \cos x = 1$$

$$\cos^3 x + \cos^2 x - \cos x - 1 = 0$$

$$\cos^2 x (\cos x + 1) - (\cos x + 1) = 0$$

$$(\cos x + 1)(\cos^2 x - 1) = 0$$

$$\cos x + 1 = 0$$

$$\cos x = -1$$

$$x = \pi$$

$$\cos^2 x - 1 = 0$$

$$\sin^2 x = 0$$

$$\sin x = 0$$

$$x = 0, \pi$$

$$a^3 + a^2 - a - 1 = 0$$

$$(a^3 + a^2) - (a + 1) = 0$$

$$a^2 \underbrace{(a+1)} - \underbrace{(a+1)} = 0$$

$$(a+1)(a^2 - 1) = 0$$

