

524: 20-23, 27-29, 92

$$20 \quad (\csc \theta + \cot \theta)(1 - \cos \theta) = \sin \theta$$

$$\left(\frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} \right) (1 - \cos \theta) =$$

$$\left(\frac{1 + \cos \theta}{\sin \theta} \right) (1 - \cos \theta) =$$

$$\left(\frac{1 - \cos^2 \theta}{\sin \theta} \right)$$

$$\frac{\sin^2 \theta}{\sin \theta} = \sin \theta = \sin \theta \checkmark$$

Δ s/c

combine
fractions

simplify

Pythag.

$$21. \quad \sin^2 \theta \tan^2 \theta = \tan^2 \theta - \sin^2 \theta$$

$$\sin^2 \theta \cdot \frac{\sin^2 \theta}{\cos^2 \theta} \quad \star$$

Stuck!

$$\frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta$$

combine



$$\frac{\sin^2 \theta - \sin^2 \theta \cos^2 \theta}{\cos^2 \theta}$$

$$\begin{matrix} x - xy \\ x(1-y) \end{matrix}$$

$$\frac{\sin^2 \theta (1 - \cos^2 \theta)}{\cos^2 \theta}$$

factor

$$\star \frac{\sin^2 \theta \cdot \sin^2 \theta}{\cos^2 \theta}$$

Pythag.

$$\sin^2 \theta \tan^2 \theta = \sin^2 \theta \tan^2 \theta \checkmark$$

$$22 \quad \frac{1 - \tan^2 \theta}{1 - \cot^2 \theta} = \frac{\cos^2 \theta - 1}{\cos^2 \theta}$$

$$\frac{1 - \left(\frac{\sin^2 \theta}{\cos^2 \theta}\right)}{1 - \left(\frac{\cos^2 \theta}{\sin^2 \theta}\right)}$$

$$\frac{\left(\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}\right)}{\left(\frac{\sin^2 \theta - \cos^2 \theta}{\sin^2 \theta}\right)} \uparrow$$

$$\frac{\sin^2 \theta (\cos^2 \theta - \sin^2 \theta)}{\cos^2 \theta (\sin^2 \theta - \cos^2 \theta)}$$

$$\frac{-\sin^2 \theta (\sin^2 \theta - \cos^2 \theta)}{\cos^2 \theta (\sin^2 \theta - \cos^2 \theta)}$$

$$= \frac{(\cos^2 \theta - 1)}{\cos^2 \theta}$$

Δ SIC

combine

flip & multiply

$$\cos^2 \theta - \sin^2 \theta = -(\sin^2 \theta - \cos^2 \theta)$$

$$= \frac{\cos^2 \theta - 1}{\cos^2 \theta - 1}$$

S=1-C

C-1=S



$$23 \quad \frac{1 + \csc \theta}{\sec \theta}$$

$$\frac{1 + \frac{1}{\sin \theta}}{\frac{1}{\cos \theta}}$$

$$\left(\frac{\sin \theta + 1}{\sin \theta} \right) \frac{\cos \theta}{1}$$

$$\frac{\cos \theta (\sin \theta + 1)}{\sin \theta}$$

$$\frac{\cos \theta \sin \theta + \cos \theta}{\sin \theta}$$

$$\cos \theta + \frac{\cos \theta}{\sin \theta}$$

$$\cos \theta + \cot \theta = \cos \theta + \cot \theta \quad \checkmark$$

$$= \cos \theta + \cot \theta$$

$$\cos \theta + \frac{\cos \theta}{\sin \theta}$$

$$\frac{\cos \theta \sin \theta + \cos \theta}{\sin \theta}$$

$$\frac{\cos \theta (\sin \theta + 1)}{\sin \theta}$$

$$27 \quad \sec \theta - \cos \theta = \tan \theta \sin \theta$$

$$\frac{1}{\cos \theta} - \cos \theta \quad \Delta \text{ s/c}$$

$$\frac{1 - \cos^2 \theta}{\cos \theta} \quad \text{combine}$$

$$\frac{\sin^2 \theta}{\cos \theta} \quad \text{pythag}$$

$$\frac{\sin \theta}{\cos \theta} \sin \theta \quad \text{separate}$$

$$\tan \theta \sin \theta = \tan \theta \sin \theta$$

$$+ + 1 = s \quad 28. \quad 1 - \tan^4 \theta = 2 \sec^2 \theta - \sec^4 \theta$$

$$(1 + \tan^2 \theta)(1 - \tan^2 \theta) \quad \text{factor}$$

$$* \sec^2 \theta (1 - \tan^2 \theta) \quad \text{pythag}$$

$$\sec^2 \theta - \sec^2 \theta \tan^2 \theta$$

$$\sec^2 \theta - \frac{1}{\cos^2 \theta} \cdot \frac{\sin^2 \theta}{\cos^2 \theta} \quad \Delta \text{ s/c}$$

$$\sec^2 \theta - \frac{(\sin^2 \theta)}{\cos^4 \theta} \quad \text{combine}$$

$$\frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^4 \theta} \quad \Delta \text{ s/c}$$

$$\frac{\cos^2 \theta - \sin^2 \theta}{\cos^4 \theta} \quad \text{combine}$$

Hmmm.....

$$2(\tan^2 \theta + 1) - \sec^4 \theta$$

$$\sec^2 \theta (2 - \sec^2 \theta)$$

$$\sec^2 \theta (1 + \underbrace{1 - \sec^2 \theta}_{-\tan^2 \theta})$$

$$\sec^2 \theta (1 - \tan^2 \theta) *$$

$$\frac{(\tan^2 \theta + 1)(1 - \tan^2 \theta)}{(1 + \tan^2 \theta)(1 - \tan^2 \theta)}$$

$$1 - \tan^4 \theta = 1 - \tan^4 \theta \checkmark$$

$$+ + 1 = s$$

$$- t \quad - s$$

$$-s + 1 = -t$$

$$1 - s = t \quad \leftarrow$$

Wrong Direction

$$29. (\csc \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$\csc^2 \theta - 2 \csc \theta \cot \theta + \cot^2 \theta \quad \text{FOIL}$$

$$\frac{1}{\sin^2 \theta} - 2 \left(\frac{\cos \theta}{\sin^2 \theta} \right) + \frac{\cos^2 \theta}{\sin^2 \theta} \quad \Delta \text{ s/c}$$

$$\frac{1 - 2 \cos \theta + \cos^2 \theta}{\sin^2 \theta}$$

Rewrite
1 fraction

$$\frac{(1 - \cos \theta)^2}{(1 - \cos^2 \theta)}$$

← outside

← inside

factor
Pythag.

$$\frac{1 - 2x + x^2}{(1 - x)^2}$$

$$\frac{(1 - \cos \theta)(1 - \cos \theta)}{(1 - \cos \theta)(1 + \cos \theta)}$$

expand
factor

$$\frac{1 - \cos \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{1 + \cos \theta} \quad \checkmark$$

$$92. \sin \theta + \cot \theta \cos \theta$$

$$\sin \theta + \frac{\cos \theta}{\sin \theta} \cos \theta$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta}$$

$$\frac{1}{\sin \theta} \quad \boxed{Q}$$