

HW 356: 23-28, 33-40, 43-44

Verify: 23.  $\frac{\sin \theta}{1 - \cos \theta} + \frac{\sin \theta}{1 + \cos \theta} = 2 \csc \theta$

$$\frac{\sin \theta (1 + \cos \theta) + \sin \theta (1 - \cos \theta)}{1 - \cos^2 \theta}$$

$$\frac{\sin \theta (1 + \cos \theta + 1 - \cos \theta)}{\sin^2 \theta}$$

$$\frac{2}{\sin \theta} = 2 \csc \theta = 2 \csc \theta \checkmark$$

24.  $\frac{\cos \theta}{\sec \theta} + \frac{\sin \theta}{\csc \theta} = 1$

$$\frac{\csc \theta \cos \theta + \sin \theta \sec \theta}{\sec \theta \csc \theta}$$

$$\frac{\left(\frac{\cos \theta}{\sin \theta}\right) + \left(\frac{\sin \theta}{\cos \theta}\right)}{\frac{1}{\cos \theta \sin \theta}}$$

$$\frac{\cos \theta \sin \theta}{\cos \theta \sin \theta}$$

$$\frac{(\cos^2 \theta + \sin^2 \theta)}{\cancel{\sin \theta \cos \theta}} \xrightarrow{1}$$

$$\frac{1}{\cancel{\cos \theta \sin \theta}} \xrightarrow{e}$$

$$= \frac{1}{1} = 1 \checkmark$$

$$25. \frac{\cot \theta}{1 + \csc \theta} + \frac{1 + \csc \theta}{\cot \theta} = 2 \sec \theta$$

$$\frac{\cot^2 \theta + (1 + \csc \theta)^2}{\cot \theta (1 + \csc \theta)}$$

$$\frac{(\csc^2 \theta - 1) + (1 + \csc \theta)^2}{\cot \theta (1 + \csc \theta)}$$

$$\frac{\csc^2 \theta - 1 + 1 + 2 \csc \theta + \csc^2 \theta}{\cot \theta (1 + \csc \theta)}$$

$$\frac{2 \csc^2 \theta + 2 \csc \theta}{\cot \theta (1 + \csc \theta)} \rightarrow \frac{2(\csc \theta)(\csc \theta + 1)}{\cot \theta (1 + \csc \theta)}$$

~~$$\frac{2(\cot^2 \theta + 1) + 2 \csc \theta}{\cot \theta (1 + \csc \theta)}$$~~

$$\frac{2 \cot^2 \theta + 2 + 2 \csc \theta}{\cot \theta (1 + \csc \theta)}$$

$$\frac{2\left(\frac{1}{\sin \theta}\right)}{\frac{\cos \theta}{\sin \theta}} = 2\left(\frac{1}{\cos \theta}\right) = 2 \sec \theta \checkmark$$

$$26. \frac{\cos \theta}{1 - \sin \theta} = \frac{1 + \sin \theta}{\cos \theta}$$

$$\frac{\cos \theta (1 + \sin \theta)}{(1 - \sin \theta)(1 + \sin \theta)}$$

$$\frac{\cos \theta (1 + \sin \theta)}{1 - \sin^2 \theta}$$

$$\frac{1 + \sin \theta}{\cos \theta} \checkmark$$

$$27. \frac{\cot^2 \theta}{1 + \csc \theta} = \csc \theta - 1$$

$$\frac{\csc^2 \theta - 1}{1 + \csc \theta}$$

$$\frac{(\csc \theta + 1)(\csc \theta - 1)}{1 + \csc \theta}$$

$$= \csc \theta - 1 = \csc \theta - 1$$

$$28. \frac{\sec \theta}{\tan \theta} + \frac{\csc \theta}{\cot \theta} = \sec \theta + \csc \theta$$

$$\left( \frac{\frac{1}{\cos \theta}}{\frac{\sin \theta}{\cos \theta}} \right) + \left( \frac{\frac{1}{\sin \theta}}{\frac{\cos \theta}{\sin \theta}} \right) =$$

$$= \csc \theta + \sec \theta = \sec \theta + \csc \theta$$

$$33. 2 \sin x = \sqrt{2} \quad [0, 2\pi)$$

$$\sin x = \frac{\sqrt{2}}{2} \quad x = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$34. 4 \cos^2 x = 3$$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$$

$$35. \tan^2 x - 3 = 0 \quad [0, 2\pi)$$

$$\tan = \pm\sqrt{3}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$36. 9 + \cot^2 x = 12$$

$$\cot^2 x = 3$$

$$\cot x = \pm\sqrt{3}$$

$$x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$$

$$37. 2\sin^2 x = \sin x$$

$$2\sin^2 x - \sin x = 0$$

$$\sin x (2\sin x - 1) = 0$$

$$\sin x = 0$$

$$2\sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$x = 0, \pi$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

check

$$2(0)^2 = 0 \quad \checkmark$$

$$2(0)^2 = 0$$

$$2\left(\frac{1}{2}\right)^2 = \frac{1}{2} \quad \checkmark$$

$$2\left(\frac{1}{2}\right)^2 = \frac{1}{2} \quad \checkmark$$

$$38. 3\cos x + 3 = \sin^2 x$$

$$3\cos x + 3 = 1 - \cos^2 x$$

$$\cos^2 x + 3\cos x + 2 = 0$$

$$(\cos x + 2)(\cos x + 1) = 0$$

$$\cos x = -2 \quad \cos x = -1$$

no solutions

$$x = \pi$$

$$a^2 + 3a + 2$$

$$(a+2)(a+1)$$

$$\text{check } 3(-1) + 3 = 0 \quad \checkmark$$

$$39. \sin^2 x - \sin x = 0$$

$$\sin x (\sin x - 1) = 0$$

$$\sin x = 0 \quad \sin x = 1$$

$$[0, 2\pi) \quad x = 0, \pi \quad x = \frac{\pi}{2}$$

all values

$$(-\infty, \infty)$$

$$x = 2\pi n, \pi + 2\pi n, \frac{\pi}{2} + 2\pi n$$

$$40. \tan^2 x = \tan x$$

all values

$$\tan x (\tan x - 1) = 0$$

$$\tan x = 0 \quad \tan x = 1$$

$$\text{On } [0, \pi) \quad x = 0, \pi \quad x = \frac{\pi}{4}$$

$$(-\infty, \infty)$$

$$x = 0 + \pi n, \frac{\pi}{4} + \pi n$$

$$43. \sin^2 x = 1 - \cos x$$

check

$$1 - \cos^2 x = 1 - \cos x$$

$$\cos^2 x - \cos x = 0$$

$$\cos x (\cos x - 1) = 0$$

$$\cos x = 0 \quad \cos x = 1$$

$$(1)^2 = 1 - 0 \quad \checkmark$$

$$(-1)^2 = 1 - 0 \quad \checkmark$$

$$(0)^2 = 1 - 1 \quad \checkmark$$

$$[0, 2\pi) \quad x = \frac{\pi}{2}, \frac{3\pi}{2} \quad x = 0$$

✓ ✓ ✓

$$(-\infty, \infty)$$

$$x = \frac{\pi}{2} + 2\pi n$$

$$= \frac{3\pi}{2} + 2\pi n$$

$$= 2\pi n$$

$$44. \sin x = \cos x + 1$$

$$\sin^2 x = (\cos x + 1)^2$$

$$\begin{array}{r} 1 - \cos^2 x \\ + 1 + \cos^2 x \end{array} = \begin{array}{r} \cos^2 x + 2\cos x + 1 \\ \cos^2 x \end{array} - 1$$

$$0 = 2\cos x \quad \cos x = 0$$

$$\text{On } [0, 2\pi), x = \frac{\pi}{2}, \frac{3\pi}{2}$$

Check

$$1 = 0 + 1 \checkmark$$

$$-1 = 0 + 1 \times$$

$$\text{on } (-\infty, \infty)$$

$x = \frac{\pi}{2} + 2\pi n$