

317: 22-23, 25-26, 29-30, 89

22. $\csc x \sec x - \tan x$

cot x

$$\begin{aligned} & \left(\frac{1}{\sin x}\right)\left(\frac{1}{\cos x}\right) - \left(\frac{\sin x}{\cos x}\right)\left(\frac{\sin x}{\sin x}\right) \\ &= \frac{1 - \sin^2 x}{\sin x \cos x} \\ &= \frac{\cos^2 x}{\sin x \cos x} = \frac{\cos x}{\sin x} = \boxed{\cot x} \end{aligned}$$

$$\begin{aligned} s^2 + c^2 &= 1 \\ \frac{s^2}{s^2} + \frac{c^2}{s^2} &= \frac{1}{s^2} \\ c^2 &= \frac{1 - s^2}{s^2} \end{aligned}$$

23. $\csc x - \cos x \cot x$

sin x

$$\begin{aligned} &= \frac{1}{\sin x} - \frac{\cos x (\cos x)}{\sin x} \\ &= \frac{1 - \cos^2 x}{\sin x} \\ &= \frac{\sin^2 x}{\sin x} = \boxed{\sin x} \end{aligned}$$

$$\begin{aligned} s^2 + c^2 &= 1 \\ s^2 &= 1 - c^2 \end{aligned}$$

25. $\frac{\tan x + \sin x \sec x}{\csc x \cdot \tan x}$

2 sin x

$$\begin{aligned} &= \frac{\left(\frac{\sin x}{\cos x}\right) + \sin x \left(\frac{1}{\cos x}\right)}{\left(\frac{1}{\sin x}\right)\left(\frac{\sin x}{\cos x}\right)} = \frac{\left(\frac{\sin x + \sin x}{\cos x}\right)}{\left(\frac{1}{\cos x}\right)} \\ &= \frac{2 \sin x}{\cos x} \cdot \frac{\cos x}{1} = \boxed{2 \sin x} \end{aligned}$$

$$\begin{aligned} s^2 + c^2 &= 1 \\ c^2 &= 1 - s^2 \end{aligned}$$

$$26. \frac{1 - \sin^2 x}{(\csc^2 x) - 1}$$

$$\frac{\sin^2 x}{\sin^2 x}$$

$$s^2 + c^2 = 1 \\ c^2 = 1 - s^2$$

$$= \frac{\cos^2 x}{\left(\frac{1}{\sin^2 x}\right) - 1}$$

I am not happy with this direction.

$$\csc^2 x + \cot^2 x \\ \csc^2 x - 1 = \cot^2 x$$

$$= \frac{\cos^2 x}{\cot^2 x} = \frac{\cos^2 x}{\left(\frac{\cos^2 x}{\sin^2 x}\right)} = \frac{\cos^2 x}{1} \cdot \frac{\sin^2 x}{\cos^2 x} = \boxed{\sin^2 x}$$

$$29. \frac{\sec^2 x}{\cot^2 x + 1} = \frac{\left(\frac{1}{\cos^2 x}\right)}{\left(\frac{1}{\sin^2 x}\right)}$$

$$\tan^2 x$$

$$= \frac{1}{\cos^2 x} \cdot \frac{\sin^2 x}{1} = \frac{\sin^2 x}{\cos^2 x} = \boxed{\tan^2 x}$$

$$30. \cot x - \csc^2 x \cot x$$

$$-\cot^3 x$$

$$\left(\frac{\sin^2 x}{\sin^2 x}\right) \left(\frac{\cos x}{\sin x}\right) - \left(\frac{1}{\sin^2 x}\right) \left(\frac{\cos x}{\sin x}\right)$$

$$\text{C.D.} = \sin^3 x$$

$$s^2 + c^2 = 1 \\ s^2 - 1 = -c^2 \\ \uparrow \frac{ab^2 - a}{a(b^2 - 1)}$$

$$= \frac{\cos x \sin^2 x - \cos x}{\sin^3 x} = \frac{\cos x (\sin^2 x - 1)}{\sin^3 x}$$

$$= \frac{\cos x (-\cos^2 x)}{\sin^3 x} = \frac{-\cos^3 x}{\sin^3 x} = \boxed{-\cot^3 x}$$

$$89. \frac{1 - \sin^2 \theta}{1 - \cos^2 \theta} \cdot \tan \theta = \frac{\sin \theta}{\cos \theta} \cdot \frac{\cos^2 \theta}{\sin^2 \theta}$$

$$+ = \frac{s}{c} \\ s^2 + c^2 = 1 \\ c^2 = 1 - s \\ s^2 = 1 - c$$

$$= \frac{\cos \theta}{\sin \theta} = \cot \theta \quad \boxed{B}$$

#30 Alternate $\cot x - \csc^2 x \cot$

$$\begin{aligned} &= \cot x \left(1 - \csc^2 x \right) \\ &= \cot x (-\cot^2 x) \\ &= \boxed{-\cot^3 x} \end{aligned}$$

$$\begin{aligned} \cot^2 + 1 &= \csc^2 \\ 1 - \csc^2 &= -\cot^2 \end{aligned}$$

#26 Alternate

$$\frac{1 - \sin^2 x}{\csc^2 x - 1} = \frac{\cos^2 x}{\left(\frac{1}{\sin^2 x}\right) - 1}$$

$$\begin{aligned} s^2 + c^2 &= 1 \\ c^2 &= 1 - s^2 \end{aligned}$$

$$= \frac{\cos^2 x}{\left(\frac{1 - \sin^2 x}{\sin^2 x}\right)} = \frac{\cos^2 x}{\left(\frac{\cos^2 x}{\sin^2 x}\right)} = \frac{\cos^2 x}{1} \cdot \frac{\sin^2 x}{\cos^2 x} = \boxed{\sin^2 x}$$