317: 22-23, 25-26, 29-30, 89

22. $\csc x \sec x - \tan x$ $\quad \underline{\cot x}$

$$\left(\frac{1}{\sin x}\right)\left(\frac{1}{\cos}\right) - \left(\frac{\sin x}{\cos x}\right)\left(\frac{\sin x}{\sin x}\right)$$

$$= \frac{1-\sin^2 x}{\sin x \cos x}$$

$s^2 + c^2 = 1$
$s^2 = 1 - c^2$
$c^2 = 1 - s^2$

$$= \frac{\cos^2 x}{\sin x \cos x} = \frac{\cos x}{\sin x} = \boxed{\cot x}$$

23. $\csc x - \cos x \cot x$ $\quad \underline{\sin x}$

$$= \frac{1}{\sin x} - \frac{\cos x}{1}\left(\frac{\cos x}{\sin x}\right)$$

$$= \frac{1-\cos^2 x}{\sin x}$$

$s^2 + c^2 = 1$
$s^2 = 1 - c^2$

$$= \frac{\sin^2 x}{\sin x} = \boxed{\sin x}$$

25. $\dfrac{\tan x + \sin x \sec x}{\csc x \cdot \tan x}$ $\quad \underline{2\sin x}$

$$= \frac{\left(\frac{\sin x}{\cos x}\right) + \sin x\left(\frac{1}{\cos x}\right)}{\left(\frac{1}{\sin x}\right)\left(\frac{\sin x}{\cos x}\right)} = \frac{\left(\frac{\sin x + \sin x}{\cos x}\right)}{\left(\frac{1}{\cos x}\right)}$$

$s^2 + c^2 = 1$
$c^2 = 1 - s^2$

$$= \frac{2\sin x}{\cos x} \cdot \frac{\cos x}{1} = \boxed{2\sin x}$$

26. $\dfrac{1-\sin^2 x}{(\csc^2 x)-1}$ ← $\sin^2 x$

$s^2+c^2=1$
$c^2=1-s^2$

$= \dfrac{\cos^2 x}{\left(\frac{1}{\sin^2 x}\right)-1}$ → I am not happy with this direction.

$\csc^2 x = 1+\cot^2 x$
$\csc^2 x - 1 = \cot^2 x$

$= \dfrac{\cos^2 x}{\cot^2 x} = \dfrac{\cos^2 x}{\left(\frac{\cos^2 x}{\sin^2 x}\right)} = \dfrac{\cos^2 x}{1}\cdot\dfrac{\sin^2 x}{\cos^2 x} = \boxed{\sin^2 x}$

29. $\dfrac{\sec^2 x}{\dfrac{\cot^2 x+1}{\csc^2 x}} = \dfrac{\left(\frac{1}{\cos^2 x}\right)}{\left(\frac{1}{\sin^2 x}\right)}$ $\tan^2 x$

$= \dfrac{1}{\cos^2 x}\cdot\dfrac{\sin^2 x}{1} = \dfrac{\sin^2 x}{\cos^2 x} = \boxed{\tan^2 x}$

30. $\cot x - \csc^2 x \cot x$ $-\cot^3 x$

$\left(\frac{\sin^2 x}{\sin^2 x}\right)\left(\frac{\cos x}{\sin x}\right)-\left(\frac{1}{\sec^2 x}\right)\left(\frac{\cos x}{\sin x}\right)$ C.D. = $\sin^3 x$

$-c^2-1$
$s^2+c^2=1$
$s^2-1=c^2$
$ab^2-a$
$a(b^2-1)$

$= \dfrac{\cos x\sin^2 x-\cos x}{\sin^3 x} = \dfrac{\cos x(\sin^2 x-1)}{\sin^3 x}$

$= \dfrac{\cos x(-\cos^2 x)}{\sin^3 x} = \dfrac{-\cos^3 x}{\sin^3 x} = \boxed{-\cot^3 x}$

89. $\dfrac{1-\sin^2\theta}{1-\cos^2\theta}\cdot\tan\theta = \dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{\cos^2\theta}{\sin^2\theta}$

$= \dfrac{\cos\theta}{\sin\theta} = \cot\theta$ B

$+=\frac{s}{c}$
$s^2+c^2=1$
$c^2=1-s$
$s^2=1-c$

#30 Alternate $\quad \cot x - \csc^2 x \cot$

$= \cot x(1 - \csc^2 x)$

$= \cot x(-\cot^2 x)$

$= \boxed{-\cot^3 x}$

$\cot^2 + 1 = \csc^2$

$1 - \csc^2 = -\cot^2$

# 26 Alternate

$$\frac{1-\sin^2 x}{\csc^2 x - 1} = \frac{\cos^2 x}{\left(\frac{1}{\sin^2 x}\right) - 1}$$

$s^2 + c^2 = 1$

$c^2 = 1 - s^2$

$$= \frac{\cos^2 x}{\left(\frac{1-\sin^2 x}{\sin^2 x}\right)} = \frac{\cos^2 x}{\left(\frac{\cos^2 x}{\sin^2 x}\right)} \quad \frac{\cos^2 x}{1} \cdot \frac{\sin^2 x}{\cos^2 x} = \boxed{\sin^2 x}$$