

Derive Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$c^2 = h^2 + x^2$$

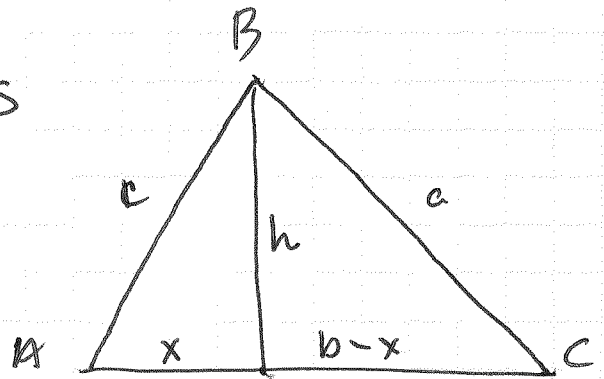
$$a^2 = h^2 + (b-x)^2$$

$$a^2 = h^2 + b^2 - 2bx + x^2$$

$$a^2 = b^2 - 2bx + \underbrace{h^2 + x^2}_{c^2}$$

$$a^2 = b^2 + c^2 - 2bx$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$



$$\cos A = \frac{x}{c}$$

$$x = c \cos A$$

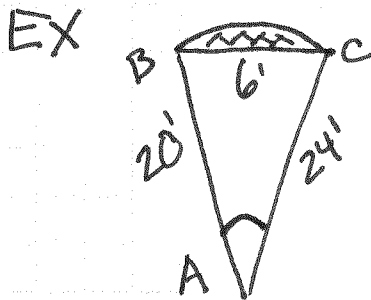
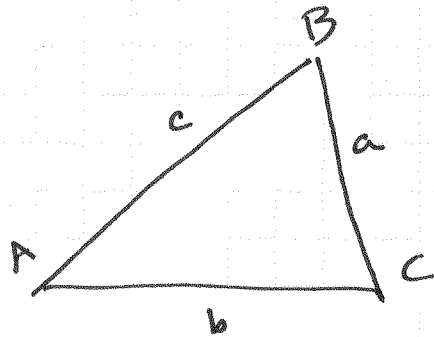
## 4.7 Law of Cosines

SAS, SSS

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$6^2 = 20^2 + 24^2 - 2(20)(24) \cos A$$

$$36 = 400 + 576 - 960 \cos A$$

$$36 = 976 - 960 \cos A$$

$$\begin{array}{r} -940 \\ -976 \end{array}$$

$$\frac{-940}{-960} = \frac{-960 \cos A}{-960}$$

$$\cos^{-1}\left(\frac{940}{960}\right) = A \quad \text{exact answer}$$

$$\boxed{A = 11.716^\circ \text{ shot angle}}$$

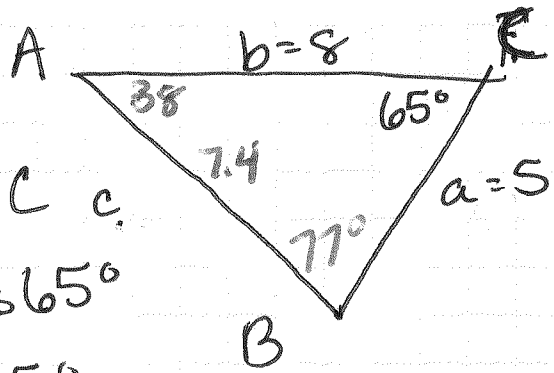
Ex 2

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 5^2 + 8^2 - 2(5)(8) \cos 65^\circ$$

$$c^2 = 25 + 64 - 80 \cos 65^\circ$$

$$\sqrt{c^2} = \sqrt{55.19} \quad \underline{c = 7.4}$$



$$\frac{\sin A}{5} = \frac{\sin 65}{7.4}$$

$$\sin A = \frac{5 \sin 65^\circ}{7.4}$$

exact answer  $\rightarrow \sin^{-1} \left( \frac{5 \sin 65^\circ}{7.4} \right) = A$

$$\underline{A = 38^\circ}$$

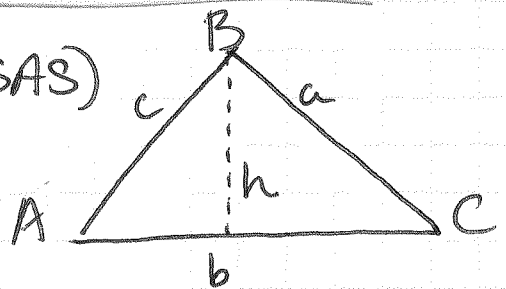
$$\angle B = 180 - (65 + 38)$$

$$= 180 - 103 = 77^\circ$$

Area of a Triangle (SAS)

$$\frac{1}{2} bh \quad \text{base} = b$$

$$h = c \sin A$$



$$\text{Area} = \frac{1}{2} b c \sin A = \frac{1}{2} a b \sin C = \frac{1}{2} a c \sin B$$

EX:  $\triangle GHT$   $g=7, h=10, J=108^\circ$

$$\text{Area} = \frac{1}{2} gh \sin J$$

$$\frac{1}{2} (10)(7) \sin 108^\circ = 33.3$$

