4.6 Composition of functions

$f(g(x))$ work inside to outside

↑ Domain of $g(x)$

Composition of functions & their inverses:

if x is in the domain of both $f(x)$ and $f^{-1}(x)$ then

$$f(f^{-1}(x)) = x \qquad f^{-1}(f(x)) = x$$

$f(f^{-1}(x)) = x$ when x is:	$f^{-1}(f(x)) = x$ when x is:
$-1 \le x \le 1$, then	$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, then
$\sin(\sin^{-1}(x)) = x$	$\sin^{-1}(\sin(x)) = x$
$-1 \le x \le 1$, then	$0 \le x \le \pi$, then
$\cos(\cos^{-1}(x)) = x$	$\cos^{-1}(\cos(x)) = x$
$-\infty < x < \infty$, then	$-\frac{\pi}{2} < x < \frac{\pi}{2}$, then
$\tan(\tan^{-1}(x)) = x$	$\tan^{-1}(\tan(x)) = x$

EX 6. Composition of Trig functions

① is x in the domain of the inside function?

yes → answer

if NO ② is it an angle?
can I find a coterminal angle that is in the domain

a. $\sin(\sin^{-1}(-\frac{1}{4}))$ yes $-\frac{1}{4}$ is in $[-1, 1]$

$$= -\frac{1}{4}$$

b. $\arctan(\tan\frac{7\pi}{4})$ $\frac{7\pi}{4}$ is NOT in $[-\frac{\pi}{2}, \frac{\pi}{2}]$

coterminal angle $\frac{7\pi}{4} - 2\pi = \frac{-\pi}{4}$ in ↑

$$\arctan(\tan(\tfrac{7\pi}{4})) = -\frac{\pi}{4}$$

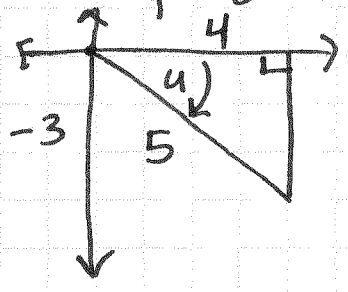
EX 7 Comp of different Trig functions

$\cos(\tan^{-1}(-\frac{3}{4}))$

① let u = inside

$u = \tan^{-1}(-\frac{3}{4})$ $\cos(u)$

$-\frac{3}{4} = \tan u$



② if u is an inverse, convert to original

③ Draw a Δ

④ Solve for the outside

$\cos u = \frac{4}{5}$

⑤ substitute out u

$$\cos(\tan^{-1}(-\tfrac{3}{4})) = \frac{4}{5}$$

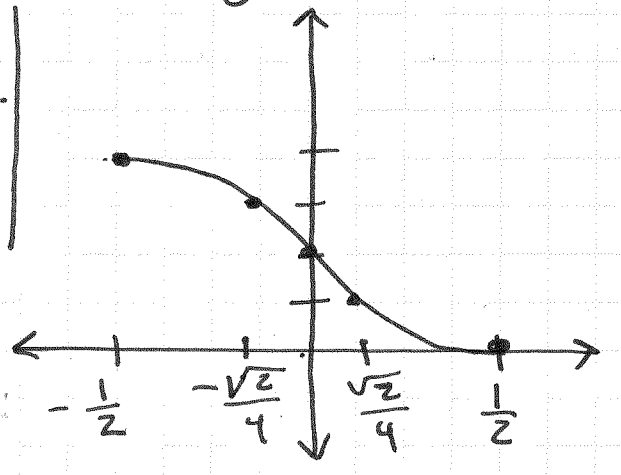
Ex 4 Graph Inverse Trig Function

Sketch graph of $y = \arccos 2x$ on $[0, \pi]$

$y = \arccos 2x \iff \cos y = 2x$

$x = \frac{1}{2}\cos y$

y	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$x = \frac{1}{2}\cos y$	$\frac{1}{2}$	$\frac{\sqrt{2}}{4}$	0	$-\frac{\sqrt{2}}{4}$	$-\frac{1}{2}$



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