

3-4 Exponential and Logarithmic Equations

One-to-one property of logs

$$\text{if } a^x = a^5, \text{ then } x = 5$$

$$\text{if } \log_b x = \log_b y \text{ then } x = y$$

$$\text{if } \log_b x = a \text{ then } b^{\log_b x} = b^a$$
$$x = b^a$$

Raising both sides of an equation to an exponent of the same base is called Exponentiation $x=y, a^x=a^y$

Example

$$\ln x = 6$$

$$e^{\ln x} = e^6$$

$$x = e^6 \approx 403.43$$

$$\text{check } \ln 403.43 \approx 6$$

← to "undo" $\ln x$, raise it to $e^{\ln x}$

"Basic"

$$\log_b b^x = x$$

Common

$$\log_{10} 10^x = x$$

natural

$$\ln e^x = x$$

$$b^{\log_b x} = x$$

$$10^{\log x} = x$$

$$e^{\ln x} = x$$

$$\frac{6 + 2 \log 5x = 18}{-6 \quad -6}$$

$$2 \log 5x = 12$$

$$\log 5x = 6$$

$$\log 5x = 10^6$$

$$\frac{5x}{5} = \frac{10^6}{5}$$

$$x = 200,000$$

$$\log 25x^2 = 12$$

$$\log 25x^2 = 10^{12}$$

$$\frac{25x^2}{25} = \frac{10^{12}}{25}$$

$$\sqrt{x^2} = \sqrt{\frac{10^{12}}{25}}$$

$$x = \sqrt{\frac{10^6}{5}}$$

$$\begin{array}{r} -3x \\ -3x \quad -3x \\ \hline -2x = -6 \\ -2x \quad -2x \\ \hline \quad \quad \quad x = 3 \end{array}$$

$$\ln 403.43 = 6$$

$$x = e^6 \approx 403.43$$

$$e^{\ln x} = e^6$$

Ex 3 $\log_4 X = \log_4 3 + \log_4 (X-2)$

$\log_4 X = \log_4 3(X-2)$

get same bases or logs

$X = 3X - 6$
 $6 = 2X$
 $3 = X$

$\begin{cases} X = 3X - 6 \\ -3X = -3X \end{cases} \rightarrow -2X = -6$
 $\frac{-2X}{-2} = \frac{-6}{-2}$
 $X = 3$

Ex 4 $4^X = 13$

common/natural log both sides

$\log 4^X = \log 13$

$X \frac{\log 4}{\log 4} = \frac{\log 13}{\log 4}$

$X = \frac{\log 13}{\log 4}$ } calculator $\frac{\ln 13}{\ln 4}$

$e^{4-3X} = 6 \approx 1.85$

$\ln e^{4-3X} = \ln 6$

$4 - 3X = \ln 6 - 4$

$\frac{-3X}{-3} = \frac{(\ln 6) - 4}{-3}$

$X = \frac{(\ln 6) - 4}{-3} \approx 0.74$

HW 196 1-3, 11-14, 22, 23, 26, 27

$$6 + 2\log 5x = 18$$

-6

-6

$$2\log 5x = 12$$

$$\log 5x = 6$$

$$10^{\log 5x} = 10^6$$

$$\frac{5x}{5} = \frac{10^6}{5}$$

$$x = 200,000$$

check