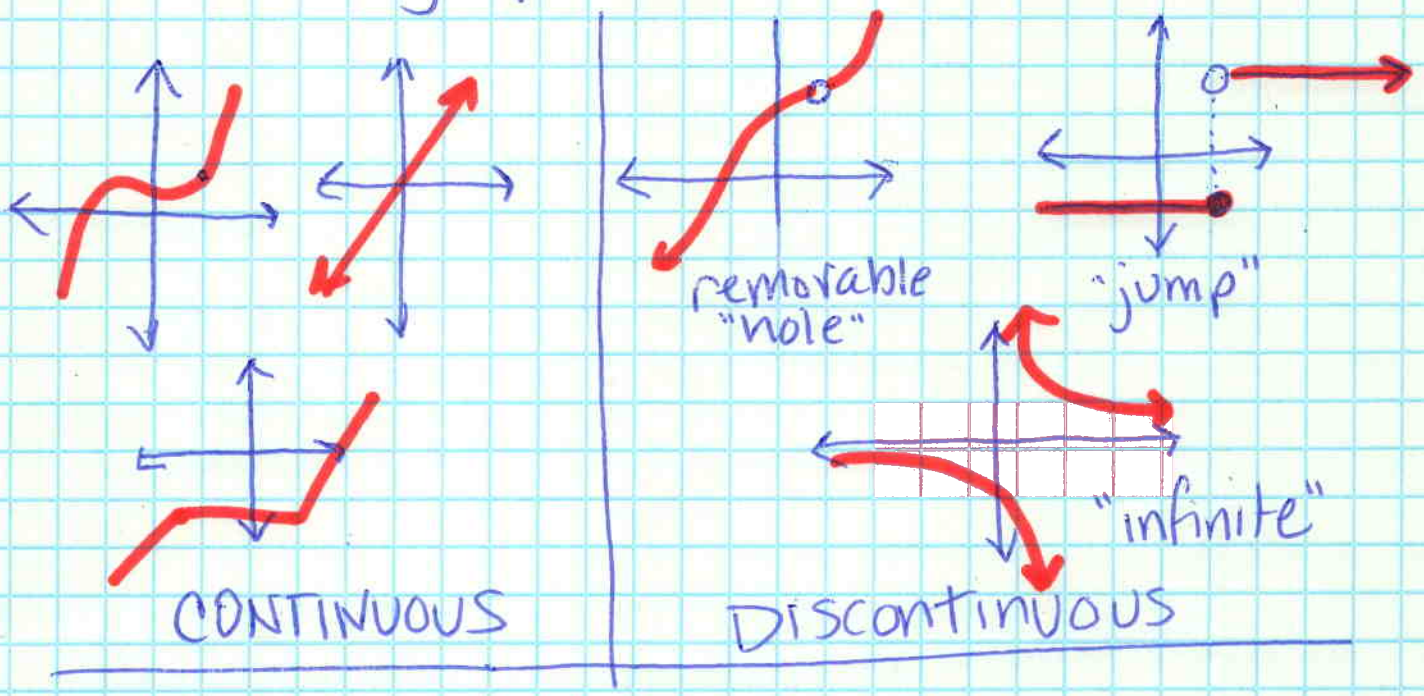
1.3 Continuity

a continuous function is one that can be drawn without lifting up the pencil.



Steps for Continuity Test at point $x=c$

1. Does $f(c)$ exist? (Plug in c)
2. Build a table, analyze the limits
$$\lim_{x \to c} f(x)$$
3. Conclusion Continuous/ Discontinuous Type?

1.3a Continuity

Continuous function: no breaks, holes, or gaps

if $f(x)$ is continuous at c, the function must approach c from the left & the right.

Limit: approaching a value, without necessisarily reaching it

$$\lim_{x \to c} f(x) = L$$

"The Limit of $f(x)$ as x approaches c is L"

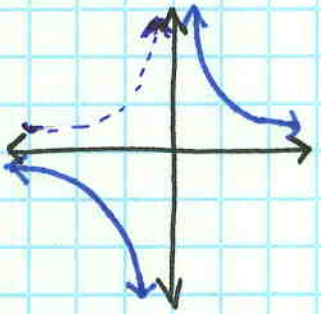
(from both directions)

Limit is a y-value

Discontinuous Functions → have some sort of "break" or <u>discontinuity</u>

Types of Discontinuity

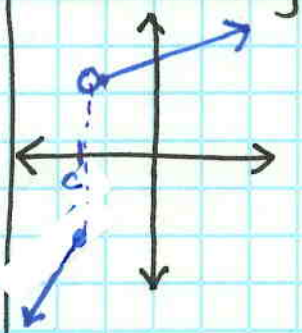
infinite discontinuity



$f(x)$ has an infinite discontinuity at $x=c$ if the function increases or decreases infinitely as $x \to c$ from the left and right

*non removable

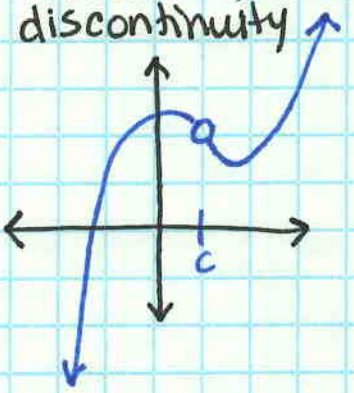
jump discontinuity



$f(x)$ has a jump discontinuity at $x=c$ if the left & right limit exist, but approach 2 different values

*non-removeable

removable discontinuity



$f(x)$ has a removable discontinuity if the function is continuous everywhere, except at $x=c$.

*the limit of $f(x)$ at c <u>exists</u>

Ex A. is $f(x)$ continuous?

$f(x) = x^2 + 5x + 6$ at $x = 3$ $(c = 3)$

1. Does $f(x)$ exist?

$f(3) = 3^2 + 5(3) + 6$

$9 + 15 + 6$

$= 30 \rightarrow$ exists & continuous jump

$x \to 3$			c			$x \leftarrow 3$
2.9	2.99	2.999	3	3.001	3.01	3.1
28.91	29.89	29.999	30	30.011	30.1101	31.11

$f(x) \longrightarrow 30 \longleftarrow f(x)$

$f(3)$ exists, and $\lim_{x \to 3} f(x) = 30$ both sides

$\therefore$ $f(x)$ is continuous

2. is $f(x)$ continuous?

$$f(x) = \begin{cases} 3x+2 & \text{if } x > 2 \\ x-1 & \text{if } x \leq 2 \end{cases} \quad \text{at } x = 2$$

1. Does $f(x)$ exist at : $x = 2$

$f(2) = x - 1$

$2 - 1 = 1$ Yes

	$x-1$	$x \to 2$		$3x+2$	$2 \leftarrow x$	
1.9	1.99	1.999	2	2.001	2.01	2.1
.9	.99	.999	1	8.003	8.03	8.3

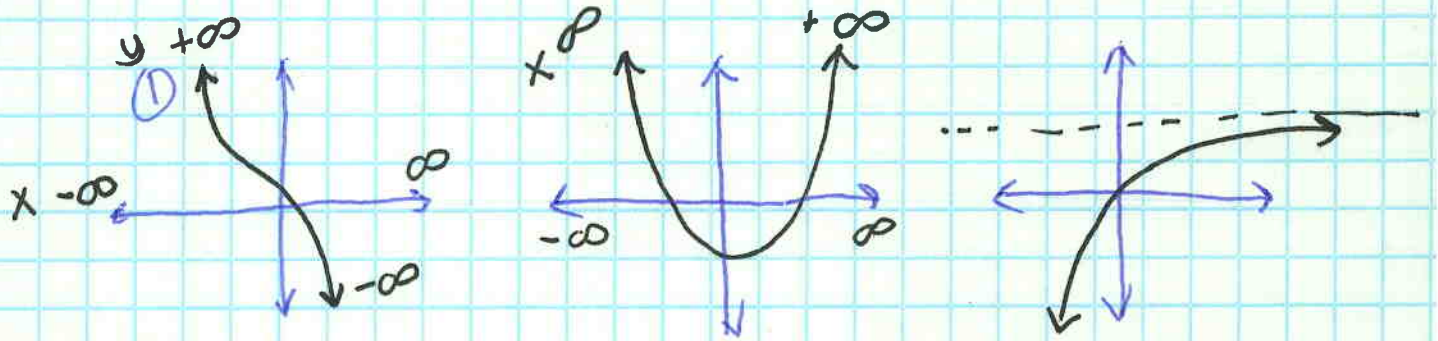
$f(x) \to 1 \neq 8 \leftarrow f(x)$

$f(x)$ has a Jump discontinuity at $x = 2$

End Behavior

What is going on as $x \to \pm\infty$

$\lim_{x \to -\infty} f(x)$ $\quad$ $\lim_{x \to +\infty} f(x)$



Ex C $f(x) = \dfrac{5x}{x-2}$

set up a table of big x values

$-\infty \leftarrow$	-1000	-100	100	1000	$x \to \infty$
$5 \leftarrow$	4.99	4.90	5.10	5.01	$\to 5$

$\lim_{x \to -\infty} f(x) = 5$, $\lim_{x \to +\infty} f(x) = 5$

y approaches 5 from the left

y approaches 5 from the right

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