## 11-2 Probability Distributions

## Classify each random variable $X$ as discrete or continuous. Explain your reasoning.

1. $X$ represents the number of text messages sent by a randomly chosen student during a given day.

SOLUTION:
Discrete; the number of text messages is countable and therefore discrete because a discrete variable can take on a finite number of possible values.

ANSWER:
Discrete; the number of text messages is countable and therefore discrete.
2. $X$ represents the time it takes a randomly selected student to complete a physics test.

SOLUTION:
Continuous; the time could be any time between a reasonable interval of time, such as between 30 and 55 minutes
ANSWER:
Continuous; the time could be any time between a reasonable interval of time, such as between 30 and 55 minutes.
3. $X$ represents the weight of a chocolate chip cookie selected at random in the school cafeteria.

SOLUTION:
Continuous; the weight can be any number and is therefore continuous because of the infinite number of possible values.

ANSWER:
Continuous; the weight can be any number.
4. $X$ represents the number of CDs owned by a student chosen at random during a given day.

SOLUTION:
Discrete; the number of CDs is countable and therefore discrete because a discrete variable can take on a finite number of possible values.

ANSWER:
Discrete; the number of CDs is countable.
5. $X$ represents the number of votes received by a candidate selected at random by a particular election.

## SOLUTION:

Discrete; the number of votes is countable and therefore discrete because a discrete variable can take on a finite number of possible values.

ANSWER:
Discrete; the number of votes is countable.

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6. $X$ represents the weight of a wrestler selected at random on a given day.

SOLUTION:
Continuous; the weight can be any number and is therefore continuous because of the infinite number of possible values.

ANSWER:
Continuous; the weight can be any number.
Construct and graph a probability distribution for each random variable $X$. Find and interpret the mean in the context of the given situation. Then find the variance and standard deviation.
7. MUSIC Students were asked how many MP3 players they own.

## Players, $X \quad$ Frequency

| 0 | 9 |
| :---: | :---: |
| 1 | 17 |
| 2 | 9 |
| 3 | 5 |
| 4 | 2 |

## SOLUTION.

To find the probability that $X$ takes on each value, divide the frequency of each value by the total number of students. The total number of students is 42 .

After making the table, graph the probability distribution with the number of players on the $x$-axis and the probability expressed as a decimal on the $y$-axis.

| Players, $\boldsymbol{X}$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{P}(\boldsymbol{X})$ | 0.21 | 0.41 | 0.21 | 0.12 | 0.05 |



To find the mean, multiply each value of $X$ by its probability $P(X)$. Then find the sum.
$0 \cdot 0.21=0$
$1 \cdot 0.41=0.41$
$2 \cdot 0.21=0.42$
$3 \cdot 0.12=0.36$
$(+) 4 \cdot 0.05=0.20$

$$
=1.39
$$

To find the variance, subtract each value of $X$ from the mean and square the difference. Then multiply each

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difference by the corresponding probability and find the sum of the products.
$(1.39-0)^{2} \cdot 0.21 \approx 0.406$
$(1.39-1)^{2} \cdot 0.41 \approx 0.062$
$(1.39-2)^{2} \cdot 0.21 \approx 0.078$
$(1.39-3)^{2} \cdot 0.12 \approx 0.311$
$(+)(1.39-4)^{2} \cdot 0.05 \approx 0.341$

$$
\approx 1.198
$$

The standard deviation is $\sqrt{1.198} \approx 1.095$ and the variance is approximately 1.20.
On average, the students have had 1 or 2 MP3 players.
ANSWER:

| Players, $\boldsymbol{X}$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{P}(\boldsymbol{X})$ | 0.21 | 0.41 | 0.21 | 0.12 | 0.05 |


1.39; Sample answer: On average, the students have had 1 or 2 MP3 players; 1.20; 1.09
8. AMUSEMENT There were 20 participants in a pie eating contest at a county fair.

\section*{| Pies Eaten, $X$ | Frequency |
| :--- | :--- |}


| 1 | 1 |
| :---: | :---: |
| 2 | 5 |
| 3 | 9 |
| 4 | 3 |
| 5 | 2 |

## SOLUTION:

To find the probability that $X$ takes on each value, divide the frequency of each value by the total number of participants, 20.

After making the table, graph the probability distribution with the number of players on the $x$-axis and the probability expressed as a decimal on the $y$-axis.

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| Pies, $\boldsymbol{X}$ | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{P}(\boldsymbol{X})$ | 0.05 | 0.25 | 0.45 | 0.15 | 0.1 |



To find the mean, multiply each value of $X$ by its probability $P(X)$. Then find the sum.

$$
1 \cdot 0.05=0
$$

$2 \cdot 0.25=0.50$
$3 \cdot 0.45=1.35$
$4 \cdot 0.15=0.60$
$(+) 5 \cdot 0.10=0.50$

$$
=2.95
$$

The mean is about 3 .
To find the variance, subtract each value of $X$ from the mean and square the difference. Then multiply each difference by the corresponding probability and find the sum of the products.
$(2.95-1)^{2} \cdot 0.05 \approx 0.190$
$(2.95-2)^{2} \cdot 0.25 \approx 0.226$
$(2.95-3)^{2} \cdot 0.45 \approx 0.001$
$(2.95-4)^{2} \cdot 0.15 \approx 0.165$
$(+)(2.95-5)^{2} \cdot 0.10 \approx 0.420$

$$
\approx 1.002
$$

The standard deviation is $\sqrt{1.002} \approx 1$ and the variance is approximately 1 .

ANSWER:

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| Pies, $\boldsymbol{X}$ | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{P}(\boldsymbol{X})$ | 0.05 | 0.25 | 0.45 | 0.15 | 0.1 |



3; Sample answer: Participants in the pie eating contest ate an average of 3 pies each; 1,1
9. BREAKFAST A sample of high school students was asked how many days they ate breakfast last week.

| Days, $\boldsymbol{X}$ | Frequency |
| :---: | :---: |
| 0 | 5 |
| 1 | 3 |
| 2 | 17 |
| 3 | 27 |
| 4 | 6 |
| 5 | 19 |
| 6 | 18 |
| 7 | 65 |

## SOLUTION:

To find the probability that $X$ takes on each value, divide the frequency of each value by the total number of students, 160.

After making the table, graph the probability distribution with the number of players on the $x$-axis and the probability expressed as a decimal on the $y$-axis. Notice that the rounded probabilities have a sum of 1.01 .

| Days, $\boldsymbol{X}$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{P}(\boldsymbol{X})$ | 0.03 | 0.02 | 0.11 | 0.17 | 0.04 | 0.12 | 0.11 | 0.41 |



To find the mean, multiply each value of $X$ by its probability $P(X)$. Then find the sum.

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$$
\begin{aligned}
0 \cdot 0.03 & =0 \\
1 \cdot 0.02 & =0.02 \\
2 \cdot 0.11 & =0.22 \\
3 \cdot 0.17 & =0.51 \\
4 \cdot 0.04 & =0.16 \\
5 \cdot 0.12 & =0.60 \\
6 \cdot 0.11 & =0.66 \\
(+) 7 \cdot 0.41 & =2.87 \\
& =5.04
\end{aligned}
$$

The mean is about 5 . The students ate breakfast an average of 5 days a week.
To find the variance, subtract each value of $X$ from the mean and square the difference. Then multiply each difference by the corresponding probability and find the sum of the products.

$$
\begin{aligned}
(5.04-0)^{2} \cdot 0.03 & \approx 0.762 \\
(5.04-1)^{2} \cdot 0.02 & \approx 0.326 \\
(5.04-2)^{2} \cdot 0.11 & \approx 1.017 \\
(5.04-3)^{2} \cdot 0.17 & \approx 0.707 \\
(5.04-4)^{2} \cdot 0.04 & \approx 0.043 \\
(5.04-5)^{2} \cdot 0.12 & \approx 0.000 \\
(5.04-6)^{2} \cdot 0.11 & \approx 0.101 \\
(+)(5.04-7)^{2} \cdot 0.41 & \approx 1.575 \\
& \approx 4.531
\end{aligned}
$$

The standard deviation is $\sqrt{4.531} \approx 2.1$ and the variance is approximately 4.5.
ANSWER:

| Days, $\boldsymbol{X}$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{P}(\boldsymbol{X})$ | 0.03 | 0.02 | 0.11 | 0.17 | 0.04 | 0.12 | 0.11 | 0.41 |



5; Sample answer: The students ate breakfast an average of 5 days a week; 4.5, 2.1

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57. REVIEW Which of the following distributions best describes the data?
$\{14,15,11,13,13,14,15,14,12,13,14,15\}$
F positively skewed
G negatively skewed
H normal
J binomial
SOLUTION:
The 11 and 12 each occur only one time, while the 13, 14, and 15 each occur at least 3 times. A graph of the distribution would have most of the data on the right and a tail on the left. This is negatively skewed.

ANSWER:
G

