11-2 Probability Distributions

Construct and graph a probability distribution for each random variable *X*. Find and interpret the mean in the context of the given situation. Then find the variance and standard deviation.

10. **HEALTH** Patients at a dentist's office were asked how many times a week they floss their teeth.

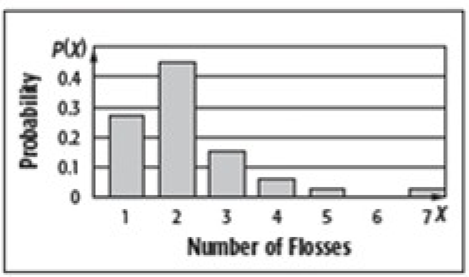
Flosses, X	Frequency
1	9
2	15
3	5
4	2
5	1
6	0
7	1

SOLUTION:

To find the probability that *X* takes on each value, divide the frequency of each value by the total number of patients, 33.

After making the table, graph the probability distribution with the number of players on the *x*-axis and the probability expressed as a decimal on the *y*-axis.

Flosses, X	1	2	3	4	5	6	7
P(X)	0.27	0.45	0.15	0.06	0.03	0.00	0.03



To find the mean, multiply each value of *X* by its probability *P*(*X*). Then find the sum.

$$\begin{aligned} 1 \cdot 0.27 &= 0.27 \\ 2 \cdot 0.45 &= 0.90 \\ 3 \cdot 0.15 &= 0.45 \\ 4 \cdot 0.06 &= 0.24 \\ 5 \cdot 0.03 &= 0.15 \\ 6 \cdot 0.00 &= 0.00 \\ (+)\,7 \cdot 0.03 &= 0.21 \\ &\approx 2.2 \end{aligned}$$

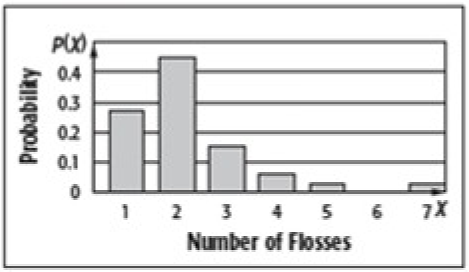
The mean is about 2. The patients flossed their teeth an average of 2 times a week.
To find the variance, subtract each value of *X* from the mean and square the difference. Then multiply each difference by the corresponding probability and find the sum of the products.

$$(2.2-1)^2 \cdot 0.27 \approx 0.326$$
$$(2.2-2)^2 \cdot 0.45 \approx 0.018$$
$$(2.2-3)^2 \cdot 0.15 \approx 0.096$$
$$(2.2-4)^2 \cdot 0.06 \approx 0.194$$
$$(2.2-5)^2 \cdot 0.03 \approx 0.235$$
$$(2.2-6)^2 \cdot 0.00 \approx 0.000$$
$$(+)(2.2-7)^2 \cdot 0.03 \approx 0.691$$
$$\approx 1.234$$

The standard deviation is $\sqrt{1.234} \approx$ 1.1 and the variance is approximately 1.2.

ANSWER:

Flosses, X	1	2	3	4	5	6	7
$P(X)$	0.27	0.45	0.15	0.06	0.03	0.00	0.03



2.2; Sample answer: The patients flossed their teeth an average of 2 times a week; 1.2, 1.1

11. **CAR INSURANCE** A car insurance policy that costs \$300 will pay \$25,000 if the car is stolen and not recovered. If the probability of a car being stolen is $p = 0.0002$, what is the expected value of the profit (or loss) to the insurance company for this policy?

SOLUTION:

The probability of the car being stolen is 0.0002. If this occurs, the insurance company loses 25,000 − 300 or \$24,700.
24,700 · 0.0002 = 4.94 loss

The probability of the car *not* being stolen is 1 − 0.0002 or 0.9998. If this occurs, the insurance company gains \$300.
300 · 0.9998 = 299.94 gain

The expected value is 299.94 − 4.94, or a \$295 gain.

ANSWER:

\$295profit

11-2 Probability Distributions

12. **FUNDRAISERS** A school hosts an annual fundraiser where raffle tickets are sold for baked goods, the values of which are shown below. Suppose 100 tickets were sold for a drawing for each of the four cakes.

What is the expected value of a participant's net gain if he or she buys a ticket for $1?

SOLUTION:

Construct a probability distribution for the possible net gains. Then find the expected value.

Gain, *X*	4	9	14	19	−1
***P*(*X*)**	0.01	0.01	0.01	0.01	0.96

Find $\sum[X \cdot P(X)]$.

$$\begin{aligned} 4 \cdot 0.01 &\approx 0.04 \\ 9 \cdot 0.01 &\approx 0.09 \\ 14 \cdot 0.01 &\approx 0.14 \\ 19 \cdot 0.01 &\approx 0.19 \\ (+)\,(-1) \cdot 0.96 &\approx -0.96 \\ &\approx -0.50 \end{aligned}$$

The expected value is a loss of $0.50.

ANSWER:

loss of $0.50

25. **GAME SHOWS** The prize wheel on a game show has 16 numbers. During one turn, a bet is made and the wheel is spun.

The payoffs for a $5 bet are shown. If a player bets $5, find the expected value for each.

Bet	Payoff	Bet	Payoff
red	$10	1	$50
green	$10	16	$50
1-4	$15	even and red	$25
5-8	$15	odd and green	$25
9-12	$15	1 or 16	$30
even	$7	odd	$7

a. green
b. even and red
c. odd

11-2 Probability Distributions

d. 1 or 16
e. 1

SOLUTION:

a. Green:

	X	$P(X)$	$X \cdot P(X)$
Win	10 − 5	$\frac{7}{16}$	$\frac{35}{16}$
Lose	−5	$\frac{9}{16}$	$-\frac{45}{16}$
Sum			$-\frac{10}{16}$

The expected value for “green” is a loss of about $0.63.

b. even and red:

	X	$P(X)$	$X \cdot P(X)$
Win	25 − 5	$\frac{3}{16}$	$\frac{60}{16}$
Lose	−5	$\frac{13}{16}$	$-\frac{65}{16}$
Sum			$-\frac{5}{16}$

The expected value for “even and red” is a loss of about $0.31.

c. odd:

	X	$P(X)$	$X \cdot P(X)$
Win	7 − 5	$\frac{1}{2}$	1
Lose	−5	$\frac{1}{2}$	−2.5
Sum			−1.5

The expected value for “odd” is a loss of $1.50.

d. 1 or 16:

	X	$P(X)$	$X \cdot P(X)$
Win	30 − 5	$\frac{1}{8}$	$\frac{25}{8}$
Lose	−5	$\frac{7}{8}$	$-\frac{35}{8}$

Sum			$-\frac{10}{8}$

The expected value for “1 or 16” is a loss of $1.25.

e. 1:

	X	$P(X)$	$X \cdot P(X)$
Win	50 − 5	$\frac{1}{16}$	$\frac{45}{16}$
Lose	−5	$\frac{15}{16}$	$-\frac{75}{16}$
Sum			$-\frac{30}{16}$

The expected value for “1” is a loss of about $1.88.

ANSWER:

a. −$0.63
b. −$0.31
c. −$1.50
d. −$1.25
e. −$1.88

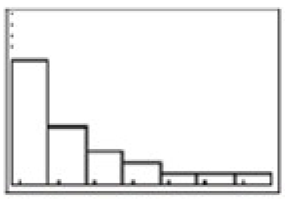
35. **ART** The prices in dollars of paintings sold at an art auction are shown.

Art Prices ($)					
1800	600	750	600	600	1800
1350	450	300	1200	750	600
750	450	2700	600	750	300
750	2300	600	450	2100	1200

a. Construct a histogram, and use it to describe the shape of the distribution.
b. Summarize the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

SOLUTION:

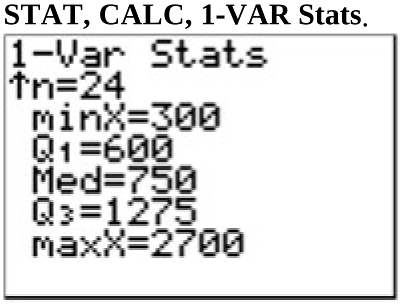
a. On a graphing calculator, press **STAT, EDIT** and input the data into L1. Then turn on **Plot1** under the **STAT PLOT** menu and choose the histogram icon. Graph using **ZoomStat** or by adjusting the window manually.



[300, 3100] scl: 400 by [0, 15] scl: 1

The tail, or lower level, extends to the right, so the graph is positively skewed. You can analyze the scale of the graph or use the **TRACE** feature to determine that the peak is in the interval [300, 700]. The graph is positively skewed, which suggests that most of the prices are on the low-end, at about $700 or less.

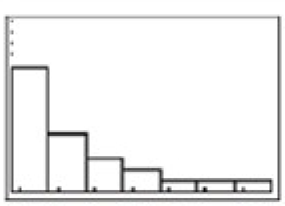
b. Since the distribution is skewed, the five-number summary can be used to describe the distribution of data. Select **STAT, CALC, 1-VAR Stats**.



The prices range from $300 to $2700 and half of the prices are between $600 and $1275.

ANSWER:

a.



[300, 3100] scl: 400 by [0, 15] scl: 1

The graph is positively skewed, which suggests that most of the prices are on the low-end, at about $700 or less.
b. The graph is positively skewed, so we can use the five-number summary. The prices range from $300 to $2700 and half of the prices are between $600 and $1275.

Graph the hyperbola given by each equation.

50. $\dfrac{(y+3)^2}{9} - \dfrac{(x+5)^2}{4} = 1$

SOLUTION:

The equation is in standard form, where $h = -5$, $k = -3$, $a = 3$, $b = 2$, and $c = \sqrt{9+4}$ or about 3.6. In the standard form of the equation, the x-term is being subtracted. Therefore, the orientation of the hyperbola is vertical.

center: $(h, k) = (-5. -3)$
vertices: $(h, k \pm a) = (-5, 0)$ and $(-5, -6)$
foci: $(h, k \pm c) = (-5, 0.6)$ and $(-5, -6.6)$

$$\textit{asymptotes}:$$

$$y - k = \pm\frac{a}{b}(x - h)$$

$$y + 3 = \frac{3}{2}(x + 5)$$

$$y = \frac{3}{2}x + \frac{3}{2}\left(5\right) - 3$$

$$y = 1.5x + 7.5 - 3$$

$$y = 1.5x + 4.5$$

$$\textit{and}$$

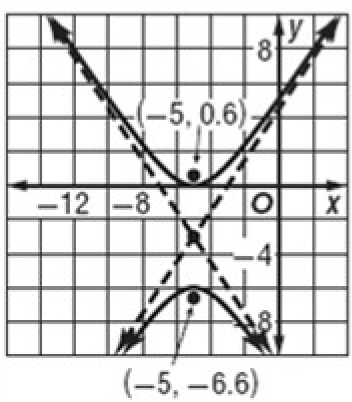
$$y + 3 = -\frac{3}{2}(x + 5)$$

$$y = -\frac{3}{2}x - \frac{3}{2}(5) - 3$$

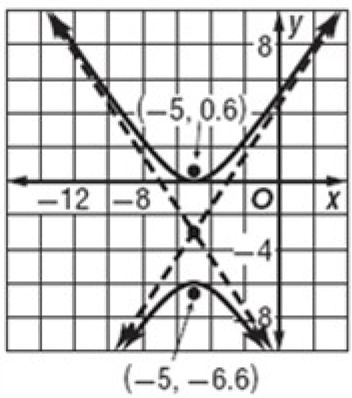
$$y = -1.5x - 7.5 - 3$$

$$y = -1.5x - 10.5$$

Graph the center, vertices, foci, and asymptotes. Then use a table of values to sketch the hyperbola.



ANSWER:



Find *AB* and *BA*, if possible.

51. $A = [2, -1]$, $B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

SOLUTION:

$A = \begin{bmatrix} 2 & -1 \end{bmatrix}$; $B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

A is a 1 × 2 matrix and *B* is a 2 × 1 matrix. Because the number of columns of *A* is equal to the number of rows of *B*, *AB* exists.

To find the first entry of *AB*, find the sum of the products of the entries in row 1 of *A* and column 1 of *B*. Follow the same procedure for row 2 column 1 of *AB*.

$$\begin{aligned} AB &= \begin{bmatrix} 2(5) + (-1)(4) \end{bmatrix} \\ &= \begin{bmatrix} 6 \end{bmatrix} \end{aligned}$$

Because the number of columns of *B* is equal to the number of rows of *A*, *BA* exists.

To find the first entry of *BA*, find the sum of the products of the entries in row 1 of *B* and column 1 of *A*. Follow the same procedure for row 2 column 1 of *BA*.

$$\begin{aligned} BA &= \begin{bmatrix} 5(2) & 5(-1) \\ 4(2) & 4(-1) \end{bmatrix} \\ &= \begin{bmatrix} 10 & -5 \\ 8 & -4 \end{bmatrix} \end{aligned}$$

ANSWER:

$AB = [6]$, $BA = \begin{bmatrix} 10 & -5 \\ 8 & -4 \end{bmatrix}$