Day 11 Probability Notes- Counting via Permutations Name: ____________________ Date: __________

## The Counting Principle:

If there are $m$ different ways of performing an operation and for each of these there are $n$ different ways of performing a second **independent** operation, then there are $mn$ different ways of performing the two operations in succession.

The word **and** suggests multiplying the possibilities.
The word **or** suggests adding the possibilities.

Example 1:

**A restaurant offers a dinner special in which a customer can select from one of 6 appetizers, a soup or salad, one of 12 entrees, and one of 8 desserts. How many different dinner specials are possible?**

multiply the number of ways for each choice.

6 × 2 × 12 × 8 = 1152 choices

app, soup salad, entree, desserts

Example 2:

**Garrett works for a bookstore. He is arranging the five best-sellers for a shelf display. If he can place the books in any order, how many different ways can Garrett arrange the books?**

5 4 3 2 1

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! = 120$ ways

Per-mutations = Every mutation (change)

Permutations: order matters

The Counting Principle can also be used to find the number of ways to arrange n objects taken in sets of r objects.
The order of the objects matters, ABC is a different arrangement than CBA.
Each arrangement is called a Permutation.

PERMUTATION FORMULAS:

| The number of permutations of $n$ objects taken $n$ at a time is $_nP_n = n!$. | The number of permutations of $n$ objects taken $r$ at a time is $_nP_r = \frac{n!}{(n-r)!}$. |
|---|---|

Example 3:

List all permutations on the symbols W, X, Y, and Z taken 4 at a time.

$_4P_4 = \frac{4!}{(4-4)!} = \frac{4\cdot3\cdot2\cdot1}{0!} = 24$ ways

$* 0! = 1$

| | | | |
|---|---|---|---|
| WXYZ | XWYZ | YWXZ | ZWXY |
| WYZX | XWZY | YWZX | ZWYX |
| WZYX | XYWZ | YXWZ | ZXWY |
| WXZY | XYZW | YXZW | ZXYW |
| WZYX | XZYW | YZWX | ZYWX |
| W~~Y~~XZ | XZWY | YZXW | ZYXW |

Example 4:

An alarm system requires a 7-digit code using the numbers 1 through 9. Each digit may only be used once.

r    n

a. How many code arrangements are possible?
b. What if it was a 3 digit code made from only the odd numbers, how many arrangements would exist?
c. How many of the 7 digit codes begin with 4?
d. How many 7 digit codes have odd numbers for their 1st three digits?

$_nP_r = {}_9P_7$

(a) $\frac{n!}{(n-r)!}$ $\frac{9!}{(9-7)!} = \frac{9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot\cancel{2\cdot1}}{\cancel{2\cdot1}} = 181{,}440$ different Combos.

(b) 5 odds 3 $_5P_3 = \frac{5!}{(5-3)!} = \frac{5\cdot4\cdot3\cdot\cancel{2!}}{\cancel{2!}} = 60$ combos of 3 odd #'s

(c) $_8P_6 = \frac{8!}{(8-6)!} = \frac{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot\cancel{2!}}{\cancel{2!}} = 20{,}160$ combo starting w/4

(d) O O O X X X X

$_5P_3 \cdot {}_6P_4$

$60 \cdot 360$

$\frac{6!}{(6-4)!} = \frac{6\cdot5\cdot4\cdot3\cdot\cancel{2!}}{\cancel{2!}}$ 21600 combos 3 odds first.