

The Counting Principle:

If there are m different ways of performing an operation and for each of these there are n different ways of performing a second **independent** operation, then there are mn different ways of performing the two operations in succession.

The word **and** suggests multiplying the possibilities.
The word **or** suggests adding the possibilities.

Example 1:

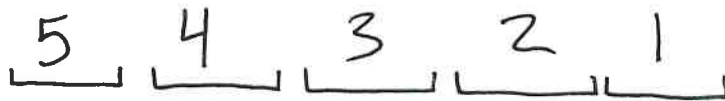
A restaurant offers a dinner special in which a customer can select from one of 6 appetizers, a soup or salad, one of 12 entrees, and one of 8 desserts. How many different dinner specials are possible?

multiply the number of ways for each choice.

$$\begin{array}{ccccccc} 6 & \times & 2 & \times & 12 & \times & 8 & = & 1152 & \text{choices} \\ \text{app} & & \begin{array}{l} \text{Soup} \\ \text{salad} \end{array} & & \text{entree} & & \text{desserts} & & & \end{array}$$

Example 2:

Garrett works for a bookstore. He is arranging the five best-sellers for a shelf display. If he can place the books in any order, how many different ways can Garrett arrange the books?



$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! = 120 \text{ ways}$$

Per-mutations = Every mutation (change)

Permutations: order matters

The Counting Principle can also be used to find the number of ways to arrange n objects taken in sets of r objects.

The order of the objects matters, ABC is a different arrangement than CBA.

Each arrangement is called a Permutation.

PERMUTATION FORMULAS:

The number of permutations of n objects taken n at a time is

$${}_n P_n = n!$$

The number of permutations of n objects taken r at a time is

$${}_n P_r = \frac{n!}{(n-r)!}$$

Example 3:

List all permutations on the symbols W, X, Y, and Z taken 4 at a time.

$${}_4 P_4 = \frac{4!}{(4-4)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{0!} = \underline{24} \text{ ways}$$

$$\ast 0! = 1$$

WXYZ	XWYZ	YWYZ	ZWXY
WYZX	XWZY	YWZY	ZWYX
WZYX	XYWZ	YXWZ	ZXWY
WXZY	XYZW	YXZW	ZXYW
WZYX	XZYW	YZWX	ZYWX
WXZY	XZWX	YZXW	ZYXW

Example 4:

An alarm system requires a r -digit code using the numbers 1 through n . Each digit may only be used once.

- How many code arrangements are possible?
- What if it was a 3 digit code made from only the odd numbers, how many arrangements would exist?
- How many of the 7 digit codes begin with 4?
- How many 7 digit codes have odd numbers for their 1st three digits?

$${}_n P_r = {}_9 P_7$$

a) $\frac{n!}{(n-r)!} = \frac{9!}{(9-7)!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2!} = 181,440$ different Combos.

b) 5 odds 3 ${}_5 P_3 = \frac{5!}{(5-3)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2!} = 60$ combos of 3 odd #'s

c) ${}_8 P_6 = \frac{8!}{(8-6)!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2!} = 20,160$ combos starting w/4

d) ${}_6 P_3 \cdot {}_6 P_4 = 60 \cdot 360 = 21,600$ combos 3 odds first.