

MIDPOINT

1. Find the midpoint of the segment with endpoints

a) (6,-7) and (3,-5)

$$\left(\frac{6+3}{2}, \frac{-7-5}{2}\right)$$

$$= \left(\frac{9}{2}, -6\right)$$

b) (12,5) and (-9,-1)

$$\left(\frac{12-9}{2}, \frac{5-1}{2}\right)$$

$$\left(\frac{3}{2}, 2\right)$$

2. Find x and y if

a) the midpoint = (3, -4) with endpoints are (x,y) and (9,14)

$$\frac{x+9}{2} = 3 \quad \frac{y+14}{2} = -4$$

$$x+9 = 6 \quad y+14 = -8$$

$$x = -3 \quad y = -22$$

(-3, -22)

b) the midpoint = (7, 10) with endpoints are (x,y) and (-5,6)

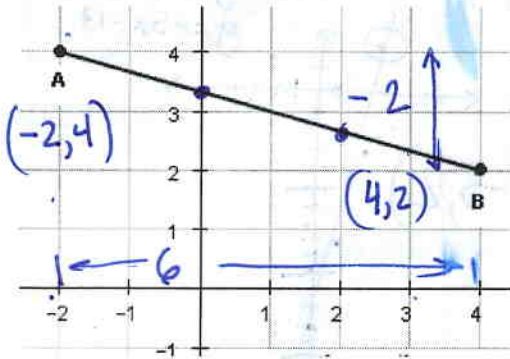
$$\frac{x-5}{2} = 7 \quad \frac{y+6}{2} = 10$$

$$x-5 = 14 \quad y+6 = 20$$

$$x = 19 \quad y = 14$$

(19, 14)

3. Find the two coordinates that divide the segment into thirds



$\frac{1}{3}$ x distance +2	add to (x,y)
$\frac{2}{3}$ x distance +4	$(-2+2, 4-\frac{2}{3})$
$\frac{1}{3}$ y distance $-\frac{2}{3}$	$(-2+4, 4-\frac{4}{3})$
$\frac{2}{3}$ y distance $-\frac{4}{3}$	$(0, 3\frac{1}{3})$
	$(2, 2\frac{2}{3})$

SLOPE, EQUATIONS, PARALLEL/ PERPENDICULAR LINES

Determine the slope of the line that contains the given points.

4. S(-1,2), W(0,4)

$$m = \frac{4-2}{0-(-1)} = 2$$

5. G(-2,5), H(1,-7)

$$m = \frac{-7-5}{1-(-2)} = \frac{-12}{3} = -4$$

6. C(0,1), D(3,3)

$$m = \frac{3-1}{3-0} = \frac{2}{3}$$

7. J(-5,-2), K(5,-4)

$$m = \frac{-4-(-2)}{5-(-5)} = \frac{-2}{10} = -\frac{1}{5}$$

Determine whether \overline{AB} and \overline{MN} are parallel, perpendicular, or neither.

8. A(0,3), B(5,-7), M(-6,7), N(-2,-1)

slopes

$$AB = \frac{-7-3}{5-0} = \frac{-10}{5} = -2 \quad MN = \frac{-1-7}{-2-(-6)} = \frac{-8}{4} = -2$$

Parallel

9. A(-1,4), B(2,-5), M(-3,2), N(3,0)

$$AB = \frac{-5-4}{2-(-1)} = \frac{-9}{3} = -3 \quad MN = \frac{0-2}{3-(-3)} = \frac{-2}{6} = -\frac{1}{3}$$

Neither

10. A(-2,-7), B(4,2), M(-2,0), N(2,6)

$$AB = \frac{2-(-7)}{4-(-2)} = \frac{9}{6} = \frac{3}{2} \quad MN = \frac{6-0}{2-(-2)} = \frac{6}{4} = \frac{3}{2}$$

Neither

11. A(-4,-83), B(4,-6), M(-3,5), N(-1,-3)

$$AB = \frac{-6-(-83)}{4-(-4)} = \frac{77}{8} \quad MN = \frac{-3-5}{-1+3} = \frac{-8}{2} = -4$$

Neither

Write an equation for each line described:

12. $m = -\frac{5}{3}, b = 0$

$$y = -\frac{5}{3}x$$

13. $m = -1, \text{ point } (0, -6)$

$$\begin{aligned} -6 &= -1(0) + b \\ b &= -6 \quad y = -1x - 6 \end{aligned}$$

14. $m = 4, \text{ point } (2, 5)$

$$\begin{aligned} 5 &= 4(2) + b \\ 3 &= b \\ y &= 4x - 3 \end{aligned}$$

15. Write an equation for the line in slope-intercept form.

a. A line parallel to $y = 3x - 2$ through $(5, 4)$

$$\begin{aligned} m &= 3 \quad 4 = 3(5) + b \\ 4 &= 15 + b \\ -11 &= b \\ y &= 3x - 11 \end{aligned}$$

b. A line perpendicular to $y = 4x + 3$ through $(-6, 3)$

$$\begin{aligned} m &= -\frac{1}{4} \quad 3 = -\frac{1}{4}(-6) + b \\ 3 &= \frac{3}{2} + b \\ +\frac{3}{2} &= b \\ y &= -\frac{1}{4}x + \frac{3}{2} \end{aligned}$$

SOLVING SYSTEMS – choose one to do with elimination, one to do with graphing and one to do with substitution

16. $\begin{cases} -2x + 6y = 6 \\ -7x + 8y = -5 \end{cases}$ (9)

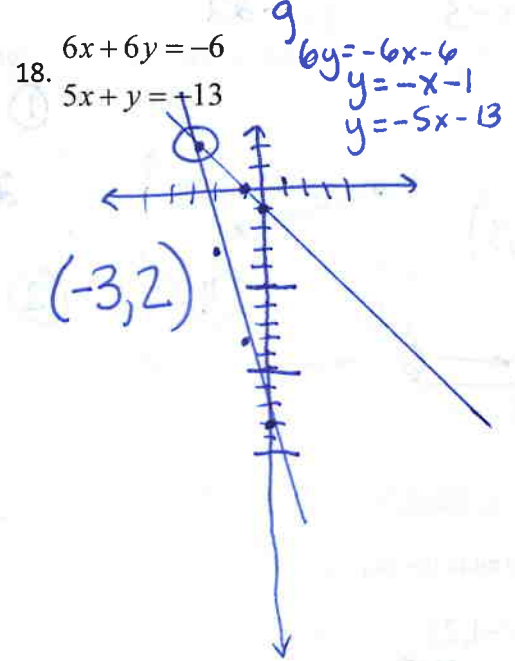
$$\begin{aligned} x - 3y &= -3 \quad x = 3y - 3 \\ -7(3y - 3) + 8y &= -5 \\ -21y + 21 + 8y &= -5 \\ -13y &= -26 \\ y &= 2 \\ -2x + 6(2) &= 6 \\ -2x + 12 &= 6 \\ -2x &= -6 \\ x &= 3 \end{aligned}$$

$$\boxed{(3, 2)}$$

17. $\begin{cases} 2x + y = 20 \\ 6x - 5y = 12 \end{cases}$ E

$$\begin{aligned} 10x + 5y &= 100 \\ 16x &= 112 \\ x &= 7 \\ 2x + y &= 20 \\ 14 + y &= 20 \\ y &= 6 \end{aligned}$$

$$\boxed{(7, 6)}$$



DISTANCE FORMULA

Find the distance between the points.

19. $(-4, -3), (1, 4)$

$$\begin{aligned} d &= \sqrt{(4 - (-3))^2 + (1 - (-4))^2} \\ d &= \sqrt{49 + 25} = \sqrt{74} \end{aligned}$$

20. $(6, -7), (3, -5)$

$$\begin{aligned} d &= \sqrt{(-5 - (-7))^2 + (3 - 6)^2} \\ &= \sqrt{4 + 9} \\ &= \sqrt{13} \end{aligned}$$

$$\boxed{\sqrt{13}}$$

21. $(-2, 3), (-1, 7)$

$$\begin{aligned} d &= \sqrt{(7 - 3)^2 + (-1 - (-2))^2} \\ &= \sqrt{16 + 1} \\ &= \sqrt{17} \end{aligned}$$

EQUATION OF A CIRCLE

Write the equation of each circle.

22. Center at the origin, radius 6

$$x^2 + y^2 = 36$$

23. Center at $(4, 3)$, radius 9

$$(x - 4)^2 + (y - 3)^2 = 81$$

23. Center at $(4, 3)$, radius 9

$$(x - 4)^2 + (y - 3)^2 = 81$$

25. C $(7, 1)$, diameter 24

$$\begin{aligned} r &= 12 \\ (x - 7)^2 + (y - 1)^2 &= 144 \end{aligned}$$

26. C $(-4, -1)$, passes through $(-2, 3)$

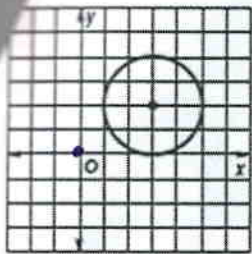
$$\begin{aligned} (x + 4)^2 + (y + 1)^2 &= r^2 \\ (-2 + 4)^2 + (3 + 1)^2 &= r^2 \\ 2^2 + 4^2 &= r^2 \\ 4 + 16 &= r^2 = 20 \end{aligned}$$

$$(x + 4)^2 + (y + 1)^2 = 20$$

27. C $(5, -2)$, passes through $(4, 0)$

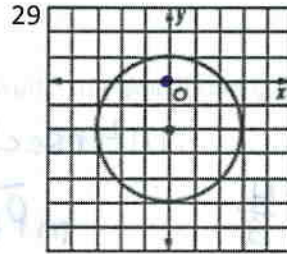
$$\begin{aligned} (x + 5)^2 + (y + 2)^2 &= r^2 \\ (4 - 5)^2 + (0 + 2)^2 &= r^2 \\ 1 + 4 &= r^2 = 5 \end{aligned}$$

$$(x - 5)^2 + (y + 2)^2 = 5$$



$$(x-3)^2 + (y-2)^2 = 4$$

$C(3, 2)$



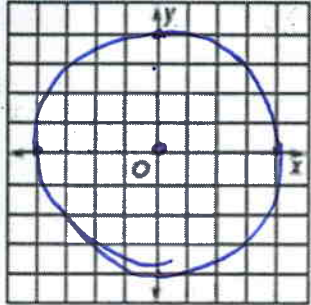
$$C = (0, -2)$$

$$(x)^2 + (y+2)^2 = 9$$

30. Find the center, the radius, and graph the circle.

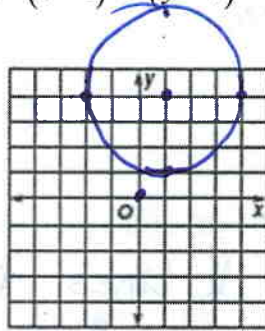
a. $x^2 + y^2 = 16$

$C(0, 0)$
 $r = 4$



b. $(x-1)^2 + (y-4)^2 = 9$

$C = (1, 4)$ $r = 3$

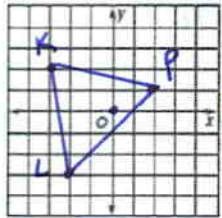


COORDINATE PROOFS

Triangles:

31. Find the measures of the sides of $\triangle KPL$ and classify each triangle by its sides.

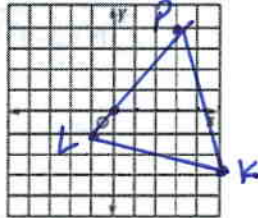
a. $K(-3, 2), P(2, 1), L(-2, -3)$



Isosceles
 $\overline{KP} \cong \overline{LK}$

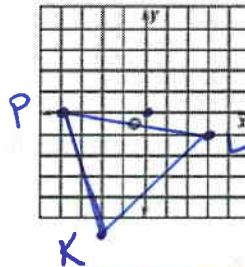
$KP = \sqrt{5^2 + 1^2} = \sqrt{26}$
 $PL = \sqrt{4^2 + 4^2} = \sqrt{32}$
 $LK = \sqrt{1^2 + 5^2} = \sqrt{26}$

b. $K(5, -3), P(3, 4), L(-1, 1)$



$KP = \sqrt{2^2 + 7^2} = \sqrt{53}$
 $PL = \sqrt{4^2 + 3^2} = \sqrt{25}$
 $LK = \sqrt{6^2 + 4^2} = \sqrt{52}$
Scalene

c. $K(-2, -6), P(-4, 0), L(3, -1)$



Isosceles
 $\overline{KP} \cong \overline{KL}$

$KP = \sqrt{2^2 + 6^2} = \sqrt{40}$
 $PL = \sqrt{7^2 + 1^2} = \sqrt{50}$
 $KL = \sqrt{5^2 + 5^2} = \sqrt{50}$

MIDSEGMENT

32. $\triangle DEF$ has vertices $D(-3, 0), E(7, -2), F(1, -4)$. Is \overline{GH} a midsegment given points $G(2, -1)$ and $H(-1, -3)$?

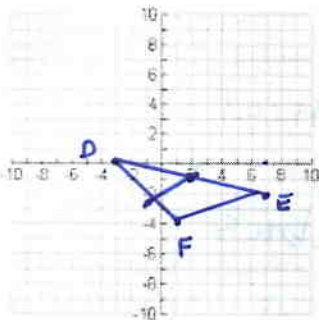
mid segment $\frac{1}{2}$ length & parallel

$d_{GH} = \sqrt{3^2 + 2^2} = \sqrt{13}$ NO

$d_{FE} = \sqrt{6^2 + 2^2} = \sqrt{40}$

$m_{GH} = \frac{(-3 - (-1))}{(-1 - 2)} = \frac{-2}{-3} = \frac{2}{3}$

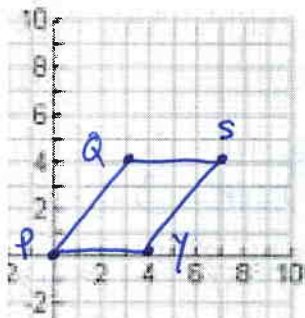
$m_{FE} = \frac{(-4 - (-2))}{(1 - 7)} = \frac{-2}{-6} = \frac{1}{3}$ NO



Quadrilaterals

33. Prove whether quadrilateral PQSY is a parallelogram. Show all work.

$P(0,0), Q(3,4), S(7,4), Y(4,0)$



$$m_{PQ} = \frac{4}{3}$$

$$m_{QS} = \frac{0}{4}$$

$$m_{SY} = \frac{-4}{-3} = \frac{4}{3}$$

$$m_{YP} = \frac{0}{-4} = 0$$

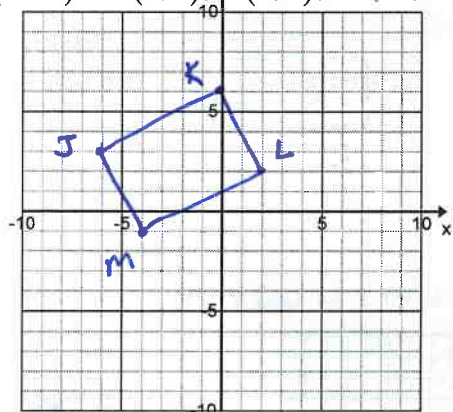
Intersecting || Lines

$$m_{\overline{PQ}} = m_{\overline{SY}} \quad \overline{PQ} \parallel \overline{SY}$$

$$m_{\overline{QS}} = m_{\overline{YP}} \quad \overline{QS} \parallel \overline{YP}$$

34. Prove whether quadrilateral JKLM is a rectangle. Justify your answer by showing all work.

$J(-6,3), K(0,6), L(2,2), M(-4,-1)$



|| Lines & \perp Lines

$$m_{JK} = \frac{3}{6} = \frac{1}{2}$$

$$m_{KL} = \frac{4}{-2} = -2$$

$$m_{LM} = \frac{3}{6} = \frac{1}{2}$$

$$m_{JM} = \frac{4}{-2} = -2$$

$$\overline{JK} \parallel \overline{LM}$$

$$\overline{KL} \parallel \overline{JM}$$

$$\overline{JK} \perp \overline{KL} \quad \angle JKL = 90^\circ$$

$$\overline{JK} \perp \overline{JM} \quad \angle KJM = 90^\circ$$

$$\overline{ML} \perp \overline{KL} \quad \angle MLK = 90^\circ$$

$$\overline{JM} \perp \overline{ML} \quad \angle JML = 90^\circ$$

Prove whether QRST is a rhombus, rectangle, or square.

35. $Q(3,5), R(3,1), S(-1,1), T(-1,5)$

$$\left. \begin{aligned} QR &= \sqrt{0+4^2} = 4 \\ RS &= \sqrt{4^2+0} = 4 \\ ST &= \sqrt{0+4^2} = 4 \\ TS &= \sqrt{4^2+0} = 4 \end{aligned} \right\} \begin{array}{l} \text{Rhombus} \\ \text{or} \\ \text{square} \end{array}$$

$$m_{QR} = \frac{4}{0} \text{ vert } \rightarrow 90^\circ$$

$$m_{RS} = \frac{0}{4} = 0 \text{ horz}$$

$$m_{ST} = \frac{4}{0} \text{ vert } \rightarrow 90^\circ$$

$$m_{QT} = \frac{0}{4} \text{ horz } \rightarrow 90^\circ$$

Square

36. $Q(-5,12), R(5,12), S(-1,4), T(-11,4)$

$$\left. \begin{aligned} \overline{QR} &= \sqrt{10^2+0} = \sqrt{100} \\ \overline{RS} &= \sqrt{6^2+8^2} = \sqrt{100} \\ \overline{ST} &= \sqrt{10^2+0} = \sqrt{100} \\ \overline{TQ} &= \sqrt{6^2+8^2} = \sqrt{100} \end{aligned} \right\} \begin{array}{l} \text{Square} \\ \text{or} \\ \text{Rhombus} \end{array}$$

$$m_{QR} = \frac{0}{10} = 0$$

$$m_{RS} = \frac{8}{6} = \frac{4}{3} \text{ Not vertical}$$

Rhombus