6-2 Matrix Multiplication, Inverses and Determinants

Given A and AB, find B.

49. $A = \begin{bmatrix} 8 & -4 \\ 3 & 6 \end{bmatrix}, AB = \begin{bmatrix} 36 & 48 \\ -24 & 48 \end{bmatrix}$

SOLUTION:

Let $B = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$.

Set up two systems of equations.

$$AB = A \cdot B$$

$$\begin{bmatrix} 36 & 48 \\ -24 & 48 \end{bmatrix} = \begin{bmatrix} 8 & -4 \\ 3 & 6 \end{bmatrix} \cdot \begin{bmatrix} w & x \\ y & z \end{bmatrix}$$

$$8w - 4y = 36$$
$$3w + 6y = -24$$
$$8x - 4z = 48$$
$$3x + 6z = 48$$

Solve for w, y, x, and z.

$$(3)(8w - 4y = 36)$$
$$(+)(-8)(3w + 6y = -24)$$

$$24w - 12y = 108$$
$$(+) -24w - 48y = 192$$
$$-60y = 300$$
$$y = -5$$
$$8w - 4(-5) = 36$$
$$8w + 20 = 36$$
$$8w = 16$$
$$w = 2$$

Solve the next system.

$$(3)(8x - 4z = 48)$$
$$(+)(-8)(3x + 6z = 48)$$

$$24x - 12z = 144$$
$$(+) -24x - 48z = -384$$
$$-60z = -240$$
$$z = 4$$
$$8x - 4(4) = 48$$
$$8x - 16 = 48$$
$$8x = 64$$
$$x = 8$$

$$B = \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ -5 & 4 \end{bmatrix}$$

50. $A = \begin{bmatrix} 5 & 0 & 1 \\ 2 & -3 & 2 \\ 1 & -1 & 4 \end{bmatrix}, AB = \begin{bmatrix} 1 & 4 \\ -16 & -6 \\ -2 & -5 \end{bmatrix}$

SOLUTION:

Matrix B must be a 3×2 matrix in order for AB to exist.

Let $B = \begin{bmatrix} u & v \\ w & x \\ y & z \end{bmatrix}$.

Set up two systems of equations.

$$AB = A \cdot B$$

$$\begin{bmatrix} 1 & 4 \\ -16 & -6 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 1 \\ 2 & -3 & 2 \\ 1 & -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} u & v \\ w & x \\ y & z \end{bmatrix}$$

$$\begin{aligned} 5u + 0w + y &= 1 \\ 2u - 3w + 2y &= -16 \\ u - w + 4y &= -2 \\ 5v + 0x + z &= 4 \\ 2v - 3x + 2z &= -6 \\ v - x + 4z &= -5 \end{aligned}$$

Solve the system.

$$\begin{aligned} & 2u - 3w + 2y = -16 \\ (+)(-3)(& u - w + 4y = -2) \end{aligned}$$

$$\begin{aligned} & 2u - 3w + 2y = -16 \\ (+) & -3u + 3w - 12y = 6 \\ & -u - 10y = -10 \end{aligned}$$

$$\begin{aligned} 5u + y &= 1 \\ y &= 1 - 5u \\ -u - 10y &= -10 \\ -u - 10(1 - 5u) &= -10 \\ -u - 10 + 50u &= -10 \\ 49u &= 0 \\ u &= 0 \\ y &= 1 - 5u \\ y &= 1 - 5(0) \\ y &= 1 \end{aligned}$$

$$\begin{aligned} u - w + 4y &= -2 \\ 0 - w + 4(1) &= -2 \\ -w &= -6 \\ w &= 6 \end{aligned}$$

Now solve the next system.

$$\begin{aligned} 2v - 3x + 2z &= -6 \\ (+)(-3)(v - x + 4z &= -5) \end{aligned}$$

$$\begin{aligned} 2v - 3x + 2z &= -6 \\ (+)\ -3v + 3x - 12z &= 15 \\ -v - 10z &= 9 \end{aligned}$$

$$\begin{aligned} 5v + z &= 4 \\ z &= 4 - 5v \\ -v - 10z &= 9 \\ -v - 10(4 - 5v) &= 9 \\ -v - 40 + 50v &= 9 \\ 49v &= 49 \\ v &= 1 \\ z &= 4 - 5v \\ z &= 4 - 5(1) \\ z &= -1 \end{aligned}$$

$$\begin{aligned} v - x + 4z &= -5 \\ 1 - x + 4(-1) &= -5 \\ -x - 3 &= -5 \\ -x &= -2 \\ x &= 2 \end{aligned}$$

$$B = \begin{bmatrix} u & v \\ w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 6 & 2 \\ 1 & -1 \end{bmatrix}$$

6-2 Matrix Multiplication, Inverses and Determinants

Find x and y.

51. $A = \begin{bmatrix} 2x & -y \\ -3y & 5x \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$, and $AB = \begin{bmatrix} -2 \\ 31 \end{bmatrix}$

SOLUTION:

$$AB = A \cdot B$$

$$\begin{bmatrix} -2 \\ 31 \end{bmatrix} = \begin{bmatrix} 2x & -y \\ -3y & 5x \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$-2 = 2x(4) + (-y)(-2)$$
$$-2 = 8x + 2y$$
$$-1 = 4x + y$$
$$-4x - 1 = y$$

$$31 = -3y(4) + 5x(-2)$$
$$31 = -12y - 10x$$
$$31 = -12(-4x - 1) - 10x$$
$$31 = 48x + 12 - 10x$$
$$19 = 38x$$
$$\frac{1}{2} = x$$

$$-4\left(\frac{1}{2}\right) - 1 = y$$
$$-2 - 1 = y$$
$$-3 = y$$