
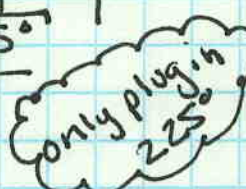


5-5-2 Half Angle Identities

- ① Half angles MUST be in $\frac{\theta}{2}$ form.
- ② Only plug in θ NOT $\frac{\theta}{2}$.
- ③ get + or - sign from the quadrant of $\frac{\theta}{2}$ NOT θ

EX 5: Exact Value of a half angle

Find $\cos 112.5^\circ$ "x.5" due to $\frac{1}{2}$ angle.

- ① get into $\frac{\theta}{2}$
multiply $\theta = \frac{2\theta}{2}$
 $\frac{2(112.5)}{2} = \frac{225^\circ}{2}$ ← leave 2 in denominator!
- ② determine \pm
 $112.5^\circ \rightarrow$ QII \cos is neg. 
- ③ use formula
 $\cos\left(\frac{225^\circ}{2}\right) = -\sqrt{\frac{1 + \cos 225^\circ}{2}}$ 
- ④ simplify (hard part)
 $-\sqrt{\frac{1 + \left(\frac{-\sqrt{2}}{2}\right)}{2}} = -\sqrt{\frac{\left(\frac{2 - \sqrt{2}}{2}\right)}{\left(\frac{2}{2}\right)}}$
- ⑤ check
 $= \sqrt{\frac{2 - \sqrt{2}}{4}} = \boxed{\frac{-\sqrt{2 - \sqrt{2}}}{2}}$ check $\cos 112.5^\circ = -.3826$
 $= \frac{-\sqrt{2 - \sqrt{2}}}{2} = -.3826$

Fraction example

$$\frac{7\pi}{12} \rightarrow \frac{\theta}{2} \quad \frac{2\left(\frac{7\pi}{12}\right)}{2} = \frac{\left(\frac{7\pi}{6}\right)}{2} = \frac{\theta}{2}$$

Ex 6 Solving using $\frac{1}{2}$ angles

$$\text{solve } \sin^2 x = 2 \cos^2\left(\frac{x}{2}\right)$$

$$\cos^2 x = (\cos x)^2$$

$$\sin^2 x = 2 \left(\cos\left(\frac{x}{2}\right) \right)^2$$

$$\sin^2 x = 2 \left(\pm \sqrt{\frac{1 + \cos x}{2}} \right)^2$$

$$= \sin^2 x = 2 \left(\frac{1 + \cos x}{2} \right) \Rightarrow \sin^2 x = 1 + \cos x$$

$$\begin{array}{r} 1 - \cos^2 x = 1 + \cos x \\ -1 + \cos^2 x = -1 + \cos^2 x \\ \hline \end{array}$$

$$0 = \cos^2 x + \cos x$$

$$0 = \cos x (\cos x + 1)$$

$$0 = \cos x$$

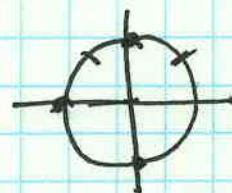
$$\cos x = 1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \pi$$

(Pythag ID)

move to one side.

factor / ZPP



check

$$\checkmark \pi$$

$$\sin^2 x = 2 \cos^2\left(\frac{x}{2}\right)$$

$$0^2 = 2(0)^2 \quad \checkmark$$

$$\checkmark \frac{\pi}{2}$$

$$1^2 = 2 \left(\frac{\sqrt{2}}{2} \right)^2 = 2 \left(\frac{2}{4} \right) \quad \checkmark$$

$$\checkmark \frac{3\pi}{2}$$

$$(-1)^2 = 2 \left(-\frac{\sqrt{2}}{2} \right)^2 = 2 \left(\frac{2}{4} \right)$$