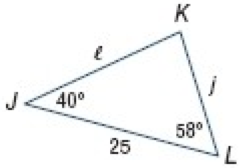
4-7 The Law of Sines and the Law of Cosines

Solve each triangle. Round to the nearest tenth, if necessary.



3.

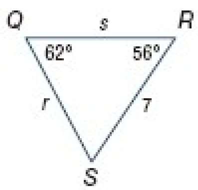
SOLUTION:

Because two angles are given, $K = 180° - (40° + 58°)$ or $82°$. Use the Law of Sines to find j and ℓ.

$$\frac{\sin J}{j} = \frac{\sin K}{k}$$
$$\frac{\sin 40°}{j} = \frac{\sin 82°}{25}$$
$$25\sin 40° = j\sin 82°$$
$$\frac{25\sin 40°}{\sin 82°} = j$$
$$16.2 \approx j$$

$$\frac{\sin L}{\ell} = \frac{\sin K}{k}$$
$$\frac{\sin 58°}{\ell} = \frac{\sin 82°}{25}$$
$$25\sin 58° = \ell\sin 82°$$
$$\frac{25\sin 58°}{\sin 82°} = \ell$$
$$21.4 \approx \ell$$

Therefore, $K = 82°$, $j \approx 16.2$, and $\ell \approx 21.4$.



4.

SOLUTION:

Because two angles are given, $S = 180° - (62° + 56°)$ or $62°$. Use the Law of Sines to find r and s.

$$\frac{\sin R}{r} = \frac{\sin Q}{Q}$$
$$\frac{\sin 56°}{r} = \frac{\sin 62°}{7}$$
$$7\sin 56° = r\sin 62°$$
$$\frac{7\sin 56°}{\sin 62°} = r$$
$$6.6 \approx r$$

$$\frac{\sin S}{s} = \frac{\sin Q}{q}$$
$$\frac{\sin 62°}{s} = \frac{\sin 62°}{7}$$
$$7\sin 62° = s\sin 62°$$
$$\frac{7\sin 62°}{\sin 62°} = s$$
$$7 \approx s$$

Therefore, $\angle S = 62°$, $r \approx 6.6$, and $s \approx 7$.

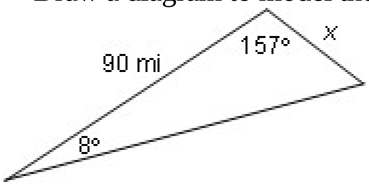
9. **TRAVEL** For the initial 90 miles of a flight, the pilot heads 8º off course in order to avoid a storm. The pilot then changes direction to head toward the destination for the remainder of the flight, making a 157° angle to the first flight course.

a. Determine the total distance of the flight.

b. Determine the distance of a direct flight to the destination

SOLUTION:

a. Draw a diagram to model the situation.

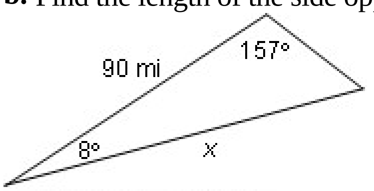


Because two angles are given, the missing angle is 180° − (157° + 8°) or 15°. Use the Law of Sines to find x.

$$\frac{\sin 8°}{x} = \frac{\sin 15°}{90}$$
$$\frac{90 \sin 8°}{x} = \sin 15°$$
$$90 \sin 8° = x \sin 15°$$
$$\frac{90 \sin 8°}{\sin 15°} = x$$
$$48.4 \approx x$$

Therefore, the distance of the flight is 90 + 48.4 or about 138.4 miles.

b. Find the length of the side opposite the 157° angle.



$$\frac{\sin 157°}{x} = \frac{\sin 15°}{90}$$
$$\frac{90 \sin 157°}{x} = \sin 15°$$
$$90 \sin 157° = x \sin 15°$$
$$\frac{90 \sin 157°}{\sin 15°} = x$$
$$135.9 \approx x$$

Therefore, the distance of a direct flight is about 135.9 miles.

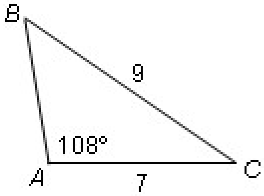
4-7 The Law of Sines and the Law of Cosines

Find all solutions for the given triangle, if possible. If no solution exists, write *no solution*. Round side lengths to the nearest tenth and angle measures to the nearest degree.

10. $a = 9, b = 7, A = 108°$

SOLUTION:

Draw a diagram of a triangle with the given dimensions.



Notice that A is obtuse and $a > b$ because 9 > 7. Therefore, one solution exists. Apply the Law of Sines to find B.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 108°}{9} = \frac{\sin B}{7}$$
$$7\sin 108° = 9\sin B$$
$$\frac{7\sin 108°}{9} = \sin B$$
$$\sin^{-1}\left(\frac{7\sin 108°}{9}\right) = B$$
$$47.71° \approx B$$

Because two angles are now known, $C \approx 180° - (108° + 48°)$ or about 24°. Apply the Law of Sines to find c.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 108°}{9} = \frac{\sin 24°}{c}$$
$$c\sin 108° = 9\sin 24°$$
$$c = \frac{9\sin 24°}{\sin 108°}$$
$$c \approx 3.85$$

Therefore, the remaining measures of $\triangle ABC$ are
$B \approx 48°$, $C \approx 24°$, and $c \approx 3.9$.

11. $a = 14, b = 15, A = 117°$

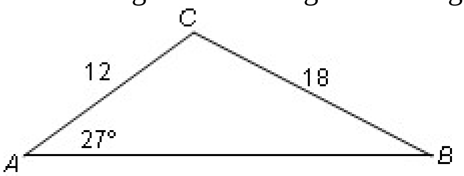
SOLUTION:

A is obtuse and $a < b$ because 14 <15. Therefore, this problem has no solution.

12. $a = 18, b = 12, A = 27°$

SOLUTION:

Draw a diagram of a triangle with the given dimensions.



Notice that A is acute and $a > b$ because 18 > 12. Therefore, one solution exists. Apply the Law of Sines to find B.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 27°}{18} = \frac{\sin B}{12}$$
$$12\sin 27° = 18\sin B$$
$$\frac{12\sin 27°}{18} = \sin B$$
$$\sin^{-1}\left(\frac{12\sin 27°}{18}\right) = B$$
$$17.62° \approx B$$

Because two angles are now known, $C \approx 180° – (27° + 17.62°)$ or about 135.38°. Apply the Law of Sines to find c.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 27°}{18} = \frac{\sin 135.38°}{c}$$
$$c\sin 27° = 18\sin 135.38°$$
$$c = \frac{18\sin 135.38°}{\sin 27°}$$
$$c \approx 27.85$$

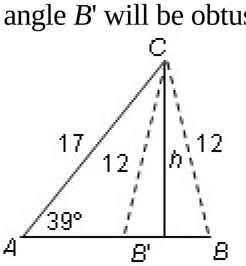
Therefore, the remaining measures of $\triangle ABC$ are
$B \approx 18°$, $C \approx 135°$, and $c \approx 27.8$.

Find two triangles with the given angle measure and side lengths. Round side lengths to the nearest tenth and angle measures to the nearest degree.

19. $A = 39°$, $a = 12$, $b = 17$

SOLUTION:

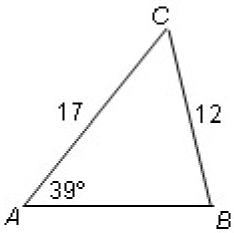
A is acute, and $h = 17 \sin 39°$ or about 10.7. Notice that $a < b$ because 12 < 17, and $a > h$ because 12 > 10.7. Therefore, two different triangles are possible with the given angle and side measures. Angle B will be acute, and angle B' will be obtuse, as shown below.



Make a reasonable sketch of each triangle and apply the Law of Sines to find each solution. Start with the case in which B is acute.

4-7 The Law of Sines and the Law of Cosines

Solution 1 $\angle B$ is acute.



Find *B*.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 39^\circ}{12} = \frac{\sin B}{17}$$
$$17 \sin 39^\circ = 12 \sin B$$
$$\frac{17 \sin 39^\circ}{12} = \sin B$$
$$0.8915 = \sin B$$
$$\sin^{-1}(0.8915) = B$$
$$63^\circ \approx B$$

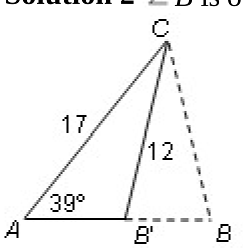
Find *C*.

$$C \approx 180^\circ - (39^\circ + 63^\circ)$$
$$\approx 78^\circ$$

Apply the Law of Sines to find *c*.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 39^\circ}{12} = \frac{\sin 78^\circ}{c}$$
$$c \sin 39^\circ = 12 \sin 78^\circ$$
$$c = \frac{12 \sin 78^\circ}{\sin 39^\circ}$$
$$c \approx 18.7$$

Solution 2 $\angle B$ is obtuse.



Note that $m\angle CB'B \cong m\angle CBB'$. To find *B*', find an obtuse angle with a sine that is also 0.8915. To do this, subtract the measure given by your calculator to nearest degree, 63º, from 180º. Therefore, *B*' is approximately 180º – 63º or 117º.

Find *C*.

$$C \approx 180^\circ - (39^\circ + 117^\circ)$$
$$\approx 24^\circ$$

Apply the Law of Sines to find *c*.

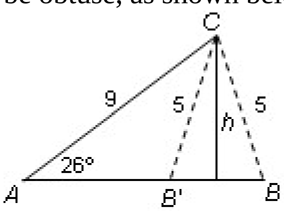
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 39°}{12} = \frac{\sin 24°}{c}$$
$$c \sin 39° = 12 \sin 24°$$
$$c = \frac{12 \sin 24°}{\sin 39°}$$
$$c \approx 7.8$$

Therefore, the missing measures for acute $\triangle ABC$ are $B \approx 63°$, $C \approx 78°$, and $c \approx 18.7$, while the missing measures for obtuse $\triangle AB'C$ are $B' \approx 117°$, $C \approx 24°$, and $c \approx 7.8$.

20. $A = 26°$, $a = 5$, $b = 9$

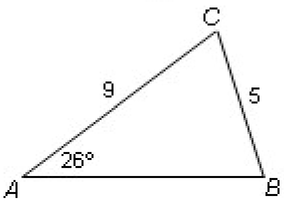
SOLUTION:

A is acute, and $h = 9 \sin 26°$ or about 3.9. Notice that $a < b$ because $5 < 9$, and $a > h$ because $5 > 3.9$. Therefore, two different triangles are possible with the given angle and side measures. Angle B will be acute, and angle B' will be obtuse, as shown below.



Make a reasonable sketch of each triangle and apply the Law of Sines to find each solution. Start with the case in which B is acute.

Solution 1 $\angle B$ is acute.



Find B.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 26°}{5} = \frac{\sin B}{9}$$
$$9 \sin 26° = 5 \sin B$$
$$\frac{9 \sin 26°}{5} = \sin B$$
$$0.2630 = \sin B$$
$$\sin^{-1}(0.2630) = B$$
$$52° \approx B$$

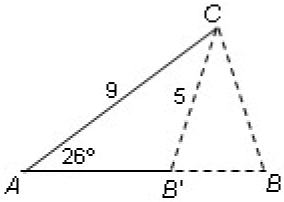
Find C.

$$C \approx 180° - (26° + 52°)$$
$$\approx 102°$$

Apply the Law of Sines to find c.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 26°}{5} = \frac{\sin 102°}{c}$$
$$c \sin 26° = 5 \sin 102°$$
$$c = \frac{5 \sin 102°}{\sin 26°}$$
$$c \approx 11.2$$

Solution 2 $\angle B$ is obtuse.



Note that $m\angle CB'B \cong m\angle CBB'$. To find *B*', find an obtuse angle with a sine that is also 0.2630. To do this, subtract the measure given by your calculator to nearest degree, 52°, from 180°. Therefore, *B*' is approximately 180° – 52° or 128°.

Find *C*.

$$C \approx 180° - (26° + 128°)$$
$$\approx 26°$$

Apply the Law of Sines to find *c*.

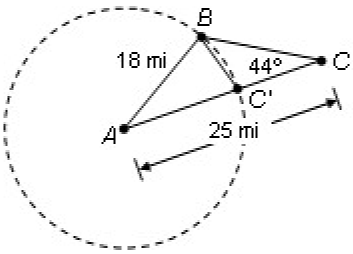
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 26°}{5} = \frac{\sin 26°}{c}$$
$$c \sin 26° = 5 \sin 26°$$
$$c = \frac{5 \sin 26°}{\sin 26°}$$
$$c = 5.0$$

Therefore, the missing measures for acute $\triangle ABC$ are
B ≈ 52°, *C* ≈ 102°, and *c* ≈ 11.2, while the missing measures for obtuse $\triangle AB'C$ are *B*' ≈ 128°, *C* ≈ 26°, and *c* ≈ 5.0.

26. **BOATING** The light from a lighthouse can be seen from an 18-mile radius. A boat is positioned so that it can just see the light from the lighthouse. A second boat is located 25 miles away from the lighthouse and is headed straight toward it, making a 44º angle with the lighthouse and the first boat. Find the distance between the two boats when the second boat enters the radius of the lighthouse light.

SOLUTION:

Draw a diagram to represent the situation.

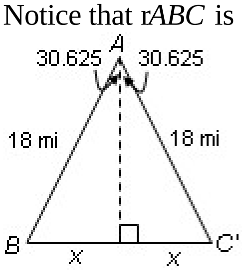


Due to the information given, use the Law of Sines to solve for *B* in r*ABC*.

$$\frac{\sin B}{AC} = \frac{\sin C}{AB}$$
$$\frac{\sin B}{25} = \frac{\sin 44°}{18}$$
$$\sin B = \frac{25 \sin 44°}{18}$$
$$B = \sin^{-1}\left(\frac{25 \sin 44°}{18}\right)$$
$$B \approx 74.75°$$

So, $A \approx 180º – (44º + 74.75º)$ or about 61.25º.

Notice that r*ABC* is an isosceles triangle. Draw an altitude from vertex *A* to *BC'*.



Use the sine function to find *x*.

$$\sin 30.625 = \frac{x}{18}$$
$$9.17 \approx x$$

Therefore, the distance between the two boats when the second boat enters the radius of the lighthouse light is 2 (9.17) or about 18.3 miles.

4-7 The Law of Sines and the Law of Cosines

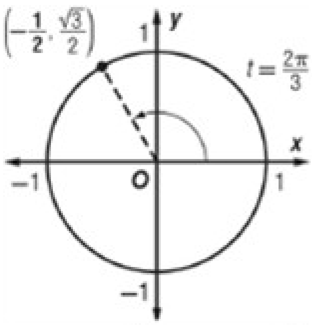
Find the exact value of each expression, if it exists.

73. $\cos^{-1} -\frac{1}{2}$

SOLUTION:

$\cos^{-1} -\frac{1}{2}$

Find a point on the unit circle on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ with a x-coordinate of $-\frac{1}{2}$.

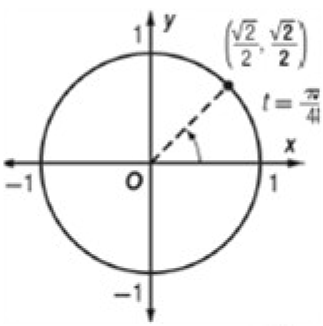


When $t = \frac{2\pi}{3}$, $\cos t = -\frac{1}{2}$. Therefore, $\cos^{-1} -\frac{1}{2} = \frac{2\pi}{3}$.

74. $\sin^{-1} \frac{\sqrt{2}}{2}$

SOLUTION:

Find a point on the unit circle on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ with a y-coordinate of $\frac{\sqrt{2}}{2}$.



When $t = \frac{\pi}{4}$, $\sin t = \frac{\sqrt{2}}{2}$. Therefore, $\sin^{-1} \frac{\sqrt{2}}{2} = \frac{\pi}{4}$.

Write a polynomial function of least degree with real coefficients in standard form that has the given zeros.

83. –1, 1, 5

SOLUTION:

Using the Linear Factorization Theorem and the zeros −1, 1, and 5, write $f(x)$ as follows.
$f(x) = a[x - (-1)][x - (1)][x - (5)]$
Let a = 1. Then write the function in standard form.

$$\begin{aligned} f(x) &= (1)(x+1)(x-1)(x-5) \\ &= (x^2-1)(x-5) \\ &= x^3 - 5x^2 - x + 5 \end{aligned}$$

Therefore, a function of least degree that has −1, 1, and 5, as zeros is $f(x) = x^3 - 5x^2 - x + 5$ or any nonzero multiple of $f(x)$.

84. –2, −0.5, 4

SOLUTION:

Using the Linear Factorization Theorem and the zeros −2, –0.5, and 4, write $f(x)$ as follows.
$f(x) = a[x - (-2)][x - (-0.5)][x - (4)]$
Let a = 1. Then write the function in standard form.

$$\begin{aligned} f(x) &= (1)(x+2)(x+0.5)(x-4) \\ &= (x^2+2.5x+1)(x-4) \\ &= x^3 - 4x^2 + 2.5x^2 - 10x + x - 4 \\ &= x^3 - 1.5x^2 - 9x - 4 \end{aligned}$$

Multiply the polynomial by 2 so that the coefficient of the x^2-term is an integer. Therefore, a function of least degree that has −2, –0.5, and 4, as zeros is $f(x) = 2x^3 - 3x^2 - 18x - 8$ or any nonzero multiple of $f(x)$.

85. –3, −2***i***, 2***i***

SOLUTION:

Using the Linear Factorization Theorem and the zeros –3, 2***i***, and −2***i***, write $f(x)$ as follows.
$f(x) = a[x - (-3)]\ [x - (2\boldsymbol{i})][x - (-2\boldsymbol{i})]$
Let a = 1. Then write the function in standard form.

$$\begin{aligned} f(x) &= (1)(x+3)(x-2\boldsymbol{i})(x+2\boldsymbol{i}) \\ &= (x+3)(x^2+4) \\ &= x^3 + 3x^2 + 4x + 12 \end{aligned}$$

Therefore, a function of least degree that has –3, 2***i***, and −2***i*** as zeros is $f(x) = x^3 + 3x^2 + 4x + 12$ or any nonzero multiple of $f(x)$.