1-3 Study Guide and Intervention

Continuity, End Behavior, and Limits

Continuity A function $f(x)$ is **continuous** at $x = c$ if it satisfies the following conditions.

(1) $f(x)$ is defined at c; in other words, $f(c)$ exists.

(2) $f(x)$ approaches the same function value to the left and right of c; in other words, $\lim_{x \to c} f(x)$ exists.

(3) The function value that $f(x)$ approaches from each side of c is $f(c)$; in other words, $\lim_{x \to c} f(x) = f(c)$.

Functions that are not continuous are **discontinuous.** Graphs that are discontinuous can exhibit **infinite discontinuity**, **jump discontinuity**, or **removable discontinuity** (also called **point discontinuity**).

Example **Determine whether each function is continuous at the given *x*-value. Justify using the continuity test. If discontinuous, identify the type of discontinuity as *infinite*, *jump*, or *removable*.**

a. $f(x) = 2|x| + 3$; $x = 2$

(1) $f(2) = 7$, so $f(2)$ exists.

(2) Construct a table that shows values for $f(x)$ for x-values approaching 2 from the left and from the right.

x	y = f(x)
1.9	6.8
1.99	6.98
1.999	6.998

x	y = f(x)
2.1	7.2
2.01	7.02
2.001	7.002

The tables show that y approaches 7 as x approaches 2 from both sides. It appears that $\lim_{x \to 2} f(x) = 7$.

(3) $\lim_{x \to 2} f(x) = 7$ and $f(2) = 7$.

The function is continuous at $x = 2$.

b. $f(x) = \dfrac{2x}{x^2 - 1}$; $x = 1$

The function is not defined at $x = 1$ because it results in a denominator of 0. The tables show that for values of x approaching 1 from the left, $f(x)$ becomes increasingly more negative. For values approaching 1 from the right, $f(x)$ becomes increasingly more positive.

x	y = f(x)
0.9	−9.5
0.99	−99.5
0.999	−999.5

x	y = f(x)
1.1	10.5
1.01	100.5
1.001	1000.5

The function has infinite discontinuity at $x = 1$.

Exercises

Determine whether each function is continuous at the given *x*-value. Justify your answer using the continuity test. If discontinuous, identify the type of discontinuity as *infinite*, *jump*, or *removable*.

1. $f(x) = \begin{cases} 2x + 1 \text{ if } x > 2 \\ x - 1 \text{ if } x \leq 2 \end{cases}$; $x = 2$

2. $f(x) = x^2 + 5x + 3$; $x = 4$

1-3 Study Guide and Intervention *(continued)*

Continuity, End Behavior, and Limits

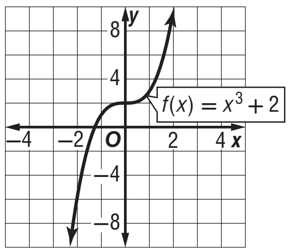
End Behavior The **end behavior** of a function describes how the function behaves at either end of the graph, or what happens to the value of $f(x)$ as x increases or decreases without bound. You can use the concept of a limit to describe end behavior.

Left-End Behavior (as x becomes more and more negative): $\lim\limits_{x \to -\infty} f(x)$

Right-End Behavior (as x becomes more and more positive): $\lim\limits_{x \to \infty} f(x)$

The $f(x)$ values may approach negative infinity, positive infinity, or a specific value.

Example **Use the graph of $f(x) = x^3 + 2$ to describe its end behavior. Support the conjecture numerically.**



As x decreases without bound, the y-values also decrease without bound. It appears the limit is negative infinity: $\lim\limits_{x \to -\infty} f(x) = -\infty$.

As x increases without bound, the y-values increase without bound. It appears the limit is positive infinity: $\lim\limits_{x \to \infty} f(x) = \infty$.

Construct a table of values to investigate function values as $|x|$ increases.

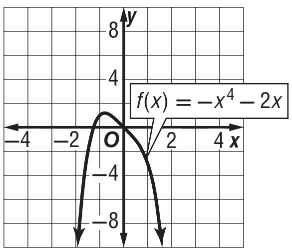
x	−1000	−100	−10	0	10	100	1000
$f(x)$	−999,999,998	−999,998	−998	2	1002	1,000,002	1,000,000,002

As $x \longrightarrow -\infty$, $f(x) \longrightarrow -\infty$. As $x \longrightarrow \infty$, $f(x) \longrightarrow \infty$. This supports the conjecture.

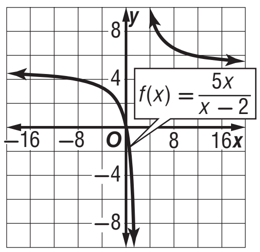
Exercises

Use the graph of each function to describe its end behavior. Support the conjecture numerically.

1.



2.



Lesson 1-3