

## 2.4 Rational Zero Theorem, Zeros of Polynomials

Real Numbers { Rational Number: fractions, integers, terminating decimals, repeating decimals  
 Irrational Numbers: non-repeating decimals, square roots,  $\pi$ ,  $e$ ,  $\sqrt{2}$ ,  $\phi$

### Rational Zero Theorem

All Possible Rational Roots

factors of  $P \leftarrow$  Constant term.  
 factors of  $q \leftarrow$  Leading coeff.

$P$  &  $q$  don't share common factors, except  $\pm 1$

\* If  $q=1$ , all possible factors are the factors of  $p$ .

Ex 1 Find all Possible Roots, factor

$$g(x) = 1x^4 + 4x^3 - 12x - 9$$

$q \uparrow$   $p \uparrow$

$P$  factors of  $P$   
 $q$   $\pm 3, \pm 1, \pm 9$

$$\begin{array}{r|rrrrr} 3 & 1 & 4 & 0 & -12 & -9 \\ & & 3 & 21 & 63 & 153 \\ \hline & 1 & 7 & 21 & 51 & \text{not zero} \end{array}$$

$$\begin{array}{r|rrrrr} 1 & 1 & 4 & 0 & -12 & -9 \\ & & 1 & 5 & 5 & -7 \\ \hline & 1 & 5 & 5 & -7 & \text{not zero} \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 1 & 4 & 0 & -12 & -9 \\ & & -1 & -3 & 3 & +9 \\ \hline & 1 & 3 & -3 & -9 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} -3 & 1 & 3 & -3 & -9 \\ & & -3 & 0 & 9 \\ \hline & 1 & 0 & -3 & 0 \\ & x^2 - 3 & & & x = \pm \sqrt{3} \text{ not rational} \end{array}$$

$(x+1)$   
 $(x+3)$   
 $(x+1)(x+3)(x^2-3)$