Lesson-by-Lesson Review

1-1 Functions (pp. 4–12)

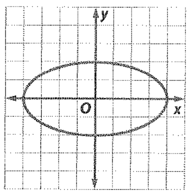
Determine whether each relation represents y as a function of x.

11. $3x - 2y = 18$ function
12. $y^3 - x = 4$ function
13.

x	y
5	7
7	9
9	11
11	13

function

14.



not a function

Let $f(x) = x^2 - 3x + 4$. Find each function value.

15. $f(5)$ 14
16. $f(-3x)$ $9x^2 + 9x + 4$

State the domain of each function.

17. $f(x) = 5x^2 - 17x + 1$ $D = \{x \mid x \in \mathbb{R}\}$
18. $g(x) = \sqrt{6x - 3}$ $D = \{x \mid x \geq 0.5, x \in \mathbb{R}\}$
19. $h(a) = \dfrac{5}{a + 5}$ $D = \{a \mid a \neq -5, a \in \mathbb{R}\}$
20. $v(x) = \dfrac{x}{x^2 - 4}$ $D = \{x \mid x \neq \pm 2, x \in \mathbb{R}\}$

Example 1

Determine whether $y^2 - 8 = x$ represents y as a function of x.

Solve for y.

$y^2 - 8 = x$ Original equation

$y^2 = x + 8$ Add 8 to each side.

$y = \pm\sqrt{x + 8}$ Take the square root of each side.

This equation does not represent y as a function of x because for any x-value greater than -8, there will be two corresponding y-values.

Example 2

Let $g(x) = -3x^2 + x - 6$. Find $g(2)$.

Substitute 2 for x in the expression $-3x^2 + x - 6$.

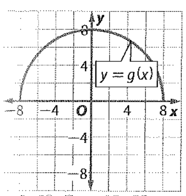
$g(2) = -3(2)^2 + 2 - 6$ $x = 2$

$= -12 + 2 - 6$ or -16 Simplify.

1-2 Analyzing Graphs of Functions and Relations (pp. 13–23)

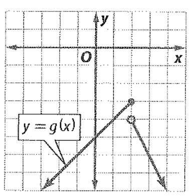
Use the graph of g to find the domain and range of each function.

21.



$D = [-8, 8]$, $R = [0, 8]$

22.



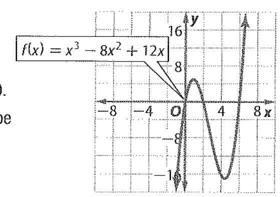
$D = \{x \mid x \in \mathbb{R}\}$, $R = (-\infty, -3)$

Find the y-intercept(s) and zeros for each function.

23. $f(x) = 4x - 9$ $-9; \frac{9}{4}$ odd
24. $f(x) = x^2 - 6x - 27$ $-27; -3, 9$
25. $f(x) = x^3 - 16x$ $0; 0, 4, -4$
26. $f(x) = \sqrt{x + 2} - 1$ $\sqrt{2} - 1; -1$

Example 3

Use the graph of $f(x) = x^3 - 8x^2 + 12x$ to find its y-intercept and zeros. Then find these values algebraically.



Estimate Graphically

It appears that $f(x)$ intersects the y-axis at $(0, 0)$, so the y-intercept is 0.

The x-intercepts appear to be at about 0, 2, and 6.

Solve Algebraically

Find $f(0)$.

$f(0) = (0)^3 - 8(0)^2 + 12(0)$ or 0

The y-intercept is 0.

Factor the related equation.

$x(x^2 - 8x + 12) = 0$

$x(x - 6)(x - 2) =$

The zeros of f are 0, 6, and 2.

Lesson-by-Lesson Review

Intervention If the given examples are not sufficient to review the topics covered by the questions, remind students that the page references tell them where to review that topic in their textbooks.

Two-Day Option Have students complete the Lesson-by-Lesson Review on pp. 77–80. Then you can use McGraw-Hill eAssessment to customize another review worksheet that practices all of the objectives of this chapter or only the objectives on which your students need more help.

Additional Answers

27. continuous at $x = 4$; The function is defined when $x = 4$. The function approaches 4 when x approaches 4 from both sides, and $f(4) = 4$.

28. continuous at $x = 10$; The function is defined when $x = 10$. The function approaches 4 when x approaches 10 from both sides

29. continuous at $x = 0$; The function is defined when $x = 0$. The function approaches 0 when x approaches 0 from both sides, and $f(0) = 0$. continuous at $x = 7$; The function is defined when $x = 7$. The function approaches 0.5 when x approaches 7 from both sides, and $f(7) = 0.5$.

30. discontinuous at $x = 2$; The function is not defined when $x = 2$. It is an infinite discontinuity. The function is continuous at $x = 4$; The function is defined when $x = 4$. The function approaches $\frac{1}{3}$ when x approaches 4 from both sides, and $f(4) = \frac{1}{3}$.

31. continuous at $x = 1$; The function is defined when $x = 1$. The function approaches 2 when x approaches 1 from both sides, and $f(1) = 2$.

32. From the graph, it appears that as $x \to \infty$, $f(x) \to -\infty$; as $x \to -\infty$, $f(x) \to \infty$.

33. From the graph, it appears that as $x \to \infty$, $f(x) \to 0$; as $x \to -\infty$, $f(x) \to 0$.

34. f is increasing on $(-\infty, -0.5)$, decreasing on $(-0.5, 0.5)$, and increasing on $(0.5, \infty)$; relative maximum at $(-0.5, 3.5)$ and relative minimum at $(0.5, 2.5)$.

35. f is decreasing on $(-\infty, -3)$, increasing on $(-3, -1.5)$, decreasing on $(-1.5, 0.5)$, and increasing on $(0.5, \infty)$; relative minimum at $(-3, 3)$, relative maximum at $(-1.5, 6)$ and relative minimum at $(0.5, -7)$.

CHAPTER 1

Study Guide and Review *Continued*

1-3 Continuity, End Behavior, and Limits (pp. 24–33)

Determine whether each function is continuous at the given x-value(s). Justify using the continuity test. If discontinuous, identify the type of discontinuity as infinite, jump, or removable. 27–31. See margin.

27. $f(x) = x^2 - 3x$; $x = 4$.

28. $f(x) = \sqrt{2x - 4}$; $x = 10$

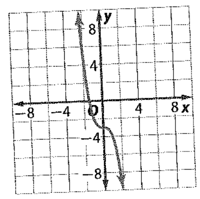
29. $f(x) = \frac{x}{x + 7}$; $x = 0$ and $x = 7$

30. $f(x) = \frac{x}{x^2 - 4}$; $x = 2$ and $x = 4$

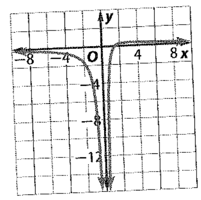
31. $f(x) = \begin{cases} 3x - 1 & \text{if } x < 1 \\ 2x & \text{if } x \geq 1 \end{cases}$; $x = 1$

Use the graph of each function to describe its end behavior. 32–33. See margin.

32.



33.



Example 4

Determine whether f ... Justify your answer u... identify the type of d...

$f(0) = -0.25$, so f is ... f gets closer to -0.2...

x	-0.1	–
$f(x)$	-0.244	–

Because $\lim_{x \to 0} f(x)$ is ... conclude that $f(x)$ is ... Because f is not defi...

Example 5

Use the graph of $f($... $-2x^4 - 5x + 1$ to ... its end behavior.

Examine the graph ...

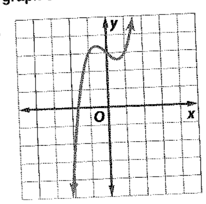
As $x \to \infty$, $f(x) \to$ ...

As $x \to -\infty$, $f(x)$ ...

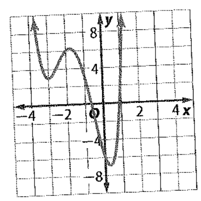
1-4 Extreme and Average Rate of Change (pp. 34–43)

Use the graph of each function to estimate intervals to the nearest 0.5 unit on which the function is increasing, decreasing, or constant. Then estimate to the nearest 0.5 unit, and classify the extrema for the graph of each function. 34–35. See margin.

34.



35.



Find the average rate of change of each function on the given interval.

36. $f(x) = -x^3 + 3x + 1$; $[0, 2]$ −1

37. $f(x) = x^2 + 2x + 5$; $[-5, 3]$ 0

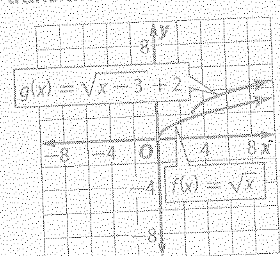
Example 6

Use the graph of ... nearest 0.5 unit ... constant. Then e... extrema for the ...

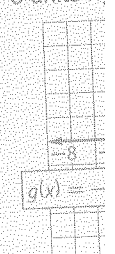
From the graph, ... that f is increasin... decreasing on (–... and increasing o...

We can estimate ... relative maximu... a relative minim...

38. $f(x) = \sqrt{x}$; $g(x)$ is the graph of $f(x)$ translated 3 units right and 2 units up.



39. $f(x) = x^2$ ... reflected ... 6 units ri...



1-5 Parent Functions and Transformations (pp. 45–55)

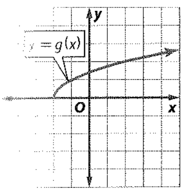
[illegible] the parent function $f(x)$ of $g(x)$, and describe how the graphs [illegible] and $f(x)$ are related. Then graph $f(x)$ and $g(x)$ on the same [illegible]. 38–41. See margin.

38. $g(x) = \sqrt{x-3} + 2$ 39. $g(x) = -(x-6)^2 - 5$

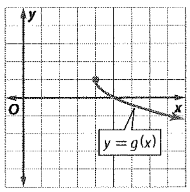
40. $g(x) = \frac{1}{2(x+7)}$ 41. $g(x) = \frac{1}{4}[\![x]\!] + 3$

[illegible] how the graphs of $f(x) = \sqrt{x}$ and $g(x)$ are related. Then [illegible] an equation for $g(x)$.

42.



43.



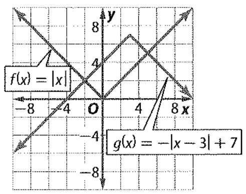
42. The graph is translated 2 units left; $g(x) = \sqrt{x+2}$.

43. The graph is reflected in the x-axis and translated 4 units right and 1 unit up; $g(x) = -\sqrt{x-4} + 1$.

Example 7

Identify the parent function $f(x)$ of $g(x) = -|x-3| + 7$, and describe how the graphs of $g(x)$ and $f(x)$ are related. Then graph $f(x)$ and $g(x)$ on the same axes.

The parent function for $g(x)$ is $f(x) = |x|$. The graph of g will be the same as the graph of f reflected in the x-axis, translated 3 units to the right, and translated 7 units up.



1-6 Function Operations and Composition of Functions (pp. 57–64)

[illegible] $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and [illegible]. State the domain of each new function. 44–47. See margin.

44. $f(x) = x + 3$, $g(x) = 2x^2 + 4x - 6$ 45. $f(x) = 4x^2 - 1$, $g(x) = 5x - 1$

46. $f(x) = x^3 - 2x^2 + 5$, $g(x) = 4x^2 - 3$ 47. $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{x^2}$

[illegible] each pair of functions, find $[f \circ g](x)$, $[g \circ f](x)$, and [illegible].

48. $f(x) = 4x - 11$; $g(x) = 2x^2 - 8$ $8x^2 - 43$; $32x^2 - 176x + 234$; -11

49. $f(x) = x^2 + 2x + 8$; $g(x) = x - 5$ $x^2 - 8x + 23$; $x^2 + 2x + 3$; 11

50. $f(x) = x^2 - 3x + 4$; $g(x) = x^2$ $x^4 - 3x^2 + 4$; $x^4 - 6x^3 + 17x^2 - 24x + 16$; 8

[illegible] $f \circ g$. 51–52. See margin.

51. $f(x) = \frac{1}{x-3}$, $g(x) = 2x - 6$ 52. $f(x) = \sqrt{x-2}$, $g(x) = 6x - 7$

Example 8

Given $f(x) = x^3 - 1$ and $g(x) = x + 7$, find $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$. State the domain of each new function.

$(f+g)(x) = f(x) + g(x)$
$= (x^3 - 1) + (x + 7)$
$= x^3 + x + 6$

The domain of $(f+g)(x)$ is $(-\infty, \infty)$.

$(f-g)(x) = f(x) - g(x)$
$= (x^3 - 1) - (x + 7)$
$= x^3 - x - 8$

The domain of $(f-g)(x)$ is $(-\infty, \infty)$.

$(f \cdot g)(x) = f(x) \cdot g(x)$
$= (x^3 - 1)(x + 7)$
$= x^4 + 7x^3 - x - 7$

The domain of $(f \cdot g)(x)$ is $(-\infty, \infty)$.

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ or $\frac{x^3 - 1}{x + 7}$

The domain of $\left(\frac{f}{g}\right)(x)$ is $D = (-\infty, -7) \cup (-7, \infty)$.

46. $(f+g)(x) = x^3 + 2x^2 + 2$; $D = (-\infty, \infty)$; $(f-g)(x) = x^3 - 6x^2 + 8$; $D = (-\infty, \infty)$; $(f \cdot g)(x) = 4x^5 - 8x^4 - 3x^3 + 26x^2 - 15$; $D = (-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{x^3 - 2x^2 + 5}{4x^2 - 3}$; $D = \left(-\infty, -\frac{\sqrt{3}}{2}\right) \cup \left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right) \cup \left(\frac{\sqrt{3}}{2}, \infty\right)$

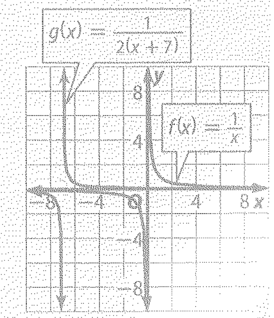
47. $(f+g)(x) = \frac{x+1}{x^2}$; $D = (-[illegible]) \cup (0, \infty)$; $(f-g)(x) = \frac{x-1}{x^2}$; $D = ([illegible], 0) \cup (0, \infty)$; $(f \cdot g)(x) = \frac{1}{x^3}$; $D = ([illegible], 0) \cup (0, \infty)$; $\left(\frac{f}{g}\right)(x) = x$; $D = ([illegible], 0) \cup (0, \infty)$

51. $[f \circ g](x) = \frac{1}{2x - 9}$ for $x \neq \frac{9}{2}$

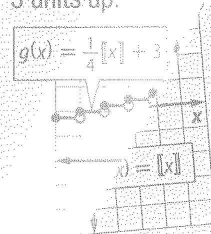
52. $[f \circ g](x) = \sqrt{6x - 9}$ for $x \geq \frac{3}{2}$

Additional Answers

40. $f(x) = \frac{1}{x}$; $g(x)$ is the graph [illegible] $f(x)$ translated 7 units left, a[illegible] compressed vertically by a fa[illegible] of $\frac{1}{2}$.



41. $f(x) = [\![x]\!]$; $g(x)$ is the g[illegible] $f(x)$ compressed vertic[illegible] factor of $\frac{1}{4}$ and trans[illegible] 3 units up.



[illegible] $2x^2 + 5x - 3$;

44. [illegible] $\infty)$; $(f-g)(x) =$ [illegible] $3x + 9$; $D = (-\infty, \infty)$; [illegible] $= 2x^3 + 10x^2 + 6x - 18$; [illegible] $(-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{1}{2(x-1)}$; $D = (-\infty, -3) \cup (-3, 1) \cup (1, \infty)$

45. $(f+g)(x) = 4x^2 + 5x - 2$; $D = (-\infty, \infty)$; $(f-g)(x) = 4x^2 - 5x$; $D = (-\infty, \infty)$; $(f \cdot g)(x) = 20x^3 - 4x^2 - 5x +$ [illegible]; $D = (-\infty, \infty)$; $\left(\frac{f}{g}\right)(x) = \frac{4x^2 - [illegible]}{5x - 1}$; $D = \left(-\infty, \frac{1}{5}\right) \cup \left(\frac{1}{5}, \infty\right)$

Anticipation Guide

Have students complete the Chapter 1 Anticipation Guide and discuss how their responses have changed now that they have completed the chapter.

Before the Test

Have students complete pp. 17 and 18 of the Study Notebook to review topics and skills presented in the chapter.

Additional Answers

57. $f^{-1}(x) = \sqrt[3]{x+2}$

58. $g^{-1}(x) = -\frac{1}{4}x + 2$

59. $h^{-1}(x) = \frac{1}{4}x^2 - 3, x \geq 0$

60. $f^{-1}(x) = \frac{-2x}{x-1}, x \neq 1$

64a. Sample answer: The number of home runs decreased, then increased and because 23 is not the smallest number of home runs.

64c. Sample answer: There were fewer home runs in 2012 than in 2007.

67a. $A(x) = 6.4516x$ cm^2

67b. $A^{-1}(x) = \frac{1}{6.4516}x$ in^2

CHAPTER 1 Study Guide and Review *Continued*

1-7 Inverse Relations and Functions (pp. 65–73)

Graph each function using a graphing calculator, and apply the horizontal line test to determine whether its inverse function exists. Write *yes* or *no.*

53. $f(x) = |x| + 6$ no

54. $f(x) = x^3$ yes

55. $f(x) = -\frac{3}{x+6}$ yes

56. $f(x) = x^3 - 4x^2$ no

Find the inverse function and state any restrictions on the domain. 57–60. See margin.

57. $f(x) = x^3 - 2$

58. $g(x) = -4x + 8$

59. $h(x) = 2\sqrt{x+3}$

60. $f(x) = \frac{x}{x+2}$

Example 9

Find the inverse function of $f(x) = \sqrt{x} - 3$ and state any restrictions on its domain.

Note that f has domain $[0, \infty)$ and range $[-3, \infty)$. Now find the inverse relation of f.

$y = \sqrt{x} - 3$ Replace $f(x)$ with y.

$x = \sqrt{y} - 3$ Interchange x and y.

$x + 3 = \sqrt{y}$ Add 3 to each side.

$(x + 3)^2 = y$ Square each side. Note that $D = (-\infty, \infty)$ and $R = [0, \infty)$.

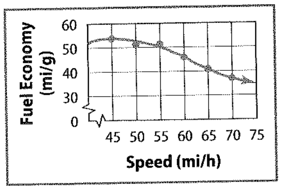
The domain of $y = (x + 3)^2$ does not equal the range of f unless restricted to $[-3, \infty)$. So, $f^{-1}(x) = (x + 3)^2$ for $x \geq -3$.

Applications and Problem Solving

61a, c. See Chapter 1 Answer Appendix.

61. CELL PHONES Basic Mobile offers a cell phone plan that charges \$39.99 per month. Included in the plan are 500 daytime minutes that can be used Monday through Friday between 7 A.M. and 7 P.M. Users are charged \$0.20 per minute for every daytime minute over 500 used. (Lesson 1-1)
   a. Write a function $p(x)$ for the cost of a month of service during which you use x daytime minutes.
   b. How much will you be charged if you use 450 daytime minutes? 550 daytime minutes? \$39.99; \$49.99
   c. Graph $p(x)$.

62. AUTOMOBILES The fuel economy for a hybrid car at various highway speeds is shown. (Lesson 1-2)



   a. Approximately what is the fuel economy for the car when traveling 50 miles per hour? Sample answer: about 51 mi/g
   b. At approximately what speed will the car's fuel economy be less than 40 miles per gallon? Sample answer: about 67 mph or faster

63. SALARIES After working for a company for five years, Ms. Washer was given a promotion. She is now earning \$1500 per month more than her previous salary. Will a function modeling her monthly income be a continuous function? Explain. (Lesson 1-3) No; sample answer: At the time of her promotion, her income had a jump discontinuity.

64. BASEBALL The table shows the number of home runs by a baseball player in each of the first 5 years he played professionally. (Lesson 1-4)

Year	2004	2005	2006	2007	2008
Number of Home Runs	5	36	23	42	42

   a. Explain why 2006 represents a relative minimum.
   b. Suppose the average rate of change of home runs between 2008 and 2011 is 5 home runs per year. How many home runs were there in 2011? 57 home runs
   c. Suppose the average rate of change of home runs between 2007 and 2012 is negative. Compare the number of home runs in 2007 and 2012. 64a, c. See margin.

65. PHYSICS A stone is thrown horizontally from the top of a cliff. The velocity of the stone measured in meters per second after t seconds can be modeled by $v(t) = -\sqrt{(9.8t)^2 + 49}$. The speed of the stone is the absolute value of its velocity. Draw a graph of the stone's speed during the first 6 seconds. (Lesson 1-5)
See Chapter 1 Answer Appendix.

66. FINANCIAL LITERACY A department store advertises \$10 off the price of any pair of jeans. How much will a pair of jeans cost if the original price is \$55 and there is 8.5% sales tax? (Lesson 1-6) \$48

67a–b. See margin.

67. MEASUREMENT One inch is approximately equal to 2.54 centimeters. (Lesson 1-7)
   a. Write a function $A(x)$ that will convert the area x of a rectangle from square inches to square centimeters.
   b. Write a function $A^{-1}(x)$ that will convert the area x of a rectangle from square centimeters to square inches.